Introduction

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Outline and prerequisites: See Syllabus

Books and additional material: See Syllabus as well. These lecture notes are far from being original, and were inspired by other lectures I myself attended, or other sets of lecture notes of friends and colleagues (some of which are available online): J. McGreevy, H. Saleur, A. Vishwanath, S. Parameswaran and A. Potter.

Overview:

Lattice model (ex: spins, interacting electrons) $\xrightarrow[\text{long distances}]{\text{low energy}}$ QFT (quantum field theory)

Examples we will encounter:

- $$S = \frac{1}{2}\int dz\,d\bar{z}\left[\chi_+\partial\chi_+ + \chi_-\bar{\partial}\chi_-\right]$$ Majorana/Ising CFT (↑ conformal)

  = Describing the critical point in $\mathbb{Z}_2$ systems in 1d

- $$H = -iv_F\int dx\left[\psi_R^\dagger\partial_x\psi_R - \psi_L^\dagger\partial_x\psi_L\right]$$ Massless Dirac Fermion in 1d

  = low energy QFT describing the spin ½ XX chain:

$$H = J\sum_i\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y\right)$$

- $H = \frac{v}{2\pi}\int dx \left[K(\partial_x \phi)^2 + K^{-1}(\partial_x \theta)^2 \right]$ Luttinger liquid (= free boson CFT)

= Describes the generic low energy physics of interacting e^- in 1d (= non-Fermi liquid)

- $\mathbb{Z}_2$ and $U(1)$ gauge theories in 2+1d (2d quantum system)

Some concepts:

- Quantum (states / operators) $\Longleftrightarrow$ Classical (path integral, partition function)
  ↳ correspondence

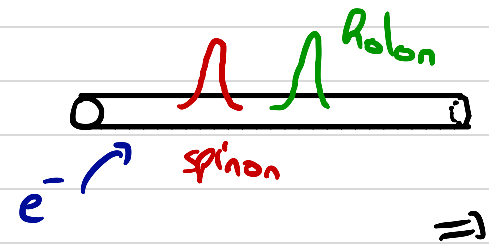
- Thermal partition function = Propagation in imaginary time

- Objects of interest: correlation functions, critical exponents

- Universality: long-distance & low-energy properties independent of microscopic details. (ex: critical exponents, topological properties etc).
  ↳ depend only on global structures: dimensionality, symmetry topological defects...

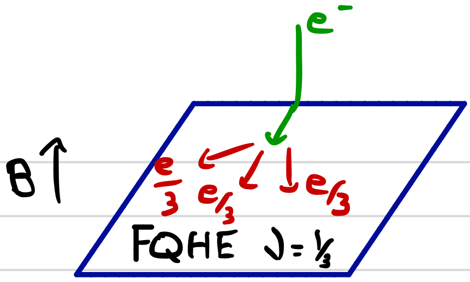
Emergence: Lorentz symmetry, conformal symmetry, SUSY, gauge invariance, fractional charge and statistics can emerge even if absent at the microscopic level.

ex



1d interacting wire

$\Rightarrow$ spin-charge separation



$\Rightarrow$ anyons and fractionalized charge

QFT: $k_B = \hbar = c = 1$ throughout

$D = d + 1$ ← time

QFT = Quantum mechanics with ∞-many degrees of freedom
$\Rightarrow$ fields $\phi(x)$ $x \in \mathbb{R}^d$

ex: strings
$$H = \int dx \, \frac{1}{2}\left[\Pi^2 + (\partial_x \phi)^2\right]$$
$$[\phi(x), \Pi(x')] = i\,\delta(x - x')$$

$\Leftrightarrow$ continuum limit of a chain of harmonic oscillators
$$H = \frac{1}{2}\sum_i \Pi_i^2 + (\phi_{i+1} - \phi_i)^2$$

Objects of interest:

- Groundstate: $|GS\rangle$ "vacuum" + low energy excitations
- $\rho = \frac{1}{Z} e^{-\beta H}$ (thermal physics)
- Dynamics: $U(t) = e^{-iHt}$ ($t = -i\beta$: thermal physics)

Typically: compute $U(\tau) = e^{-\tau H}$ then $\tau = it$ (Wick rotation)

. Correlation function: $\langle O(x,t)\, O(x',t') \dots \rangle_T$

$$\langle \dots \rangle_T = \frac{1}{Z} \mathrm{Tr}\left(e^{-\beta H} \dots\right) , \quad \text{at } T=0 \ \langle \dots \rangle = \langle GS | \dots | GS \rangle$$

e.g. magnetic system: $\langle m(x)\, m(0) \rangle \underset{x \gg a}{\sim} \begin{cases} \text{cst} & \text{Ferromagnet} \\ e^{-x/\xi} & \text{Paramagnet} \\ 1/x^{2\Delta_m} & \text{critical point} \end{cases}$

local magnetization

Cutoffs: We'll see that dealing with QFTs leads to many "infinities" that we'll have to deal with. (eg: GS energy!)

QFT = effective description valid at lengthscales $\ell \gg a$

a: UV cutoff, lattice spacing

. In particle physics: Planck scale $\ell_P \sim 10^{-35}\,m$

. Lattice models = "UV regularizations" of QFTs

. Not all QFTs can be put on a lattice: (cf. "Anomalies" "UV complete")

eg (single) chiral fermion $H = -i v_F \int dx\, \psi^\dagger \partial_x \psi$