PHY-897: Special Topics: Solid State Physics, UMass Amherst, Problem Set #3

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Due: April 25 in class

I. LUTTINGER LIQUID

Consider a generic, spinless interacting fermionic model in one dimension described at low energy by the field theory

$$H = H_0 + \int dx \left(g_2 \rho_R \rho_L + \frac{g_4}{2} (\rho_R^2 + \rho_L^2) \right),$$

with $H_0 = -iv_F \int dx (: \psi_R^\dagger \partial_x \psi_R : - : \psi_L^\dagger \partial_x \psi_L :)$ the kinetic part, $\rho_R =: \psi_R^\dagger \psi_R :$, $\rho_L =: \psi_L^\dagger \psi_L :$, and g_2 and g_4 are (not small) parameters. Compute the effective velocity and the Luttinger parameter g of the underlying Luttinger liquid theory in terms of v_F, g_2 and g_4.

II. STAGGERED MAGNETIC FIELDS IN XXZ SPIN CHAIN

Let us consider the XXZ spin$-\frac{1}{2}$ spin chain

$$H = J \sum_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z),$$

in the gapless regime $-1 < \Delta < 1$ where the low-energy physics is described by a Luttinger liquid with Luttinger parameter g. Consider a staggered magnetic field perturbation $-h_s \sum_i (-1)^i S_i^z$ to this Hamiltonian. Write down the bosonized version of this perturbation and compute its scaling dimension. Do you expect this perturbation to open up a gap in the spectrum? Discuss the nature of the groundstate in the regime where this perturbation is relevant.

III. DIMERIZED SPIN CHAINS

Let us consider a dimerized XXZ spin$-\frac{1}{2}$ spin chain where odd and even couplings are different

$$H = \sum_i J(1 + \delta(-1)^i)(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \sum_i J \Delta S_i^z S_{i+1}^z,$$

where δ is a dimerization parameter. As in the previous problem, we consider the regime $-1 < \Delta < 1$ where the low-energy physics is described by a Luttinger liquid with Luttinger parameter g.

1. First argue at the free fermion point $\Delta = 0$ that the dimerizing perturbation is relevant.
2. Now consider the case of a generic Luttinger parameter $g = \frac{\pi}{2(\pi - \arccos \Delta)}$ away from the special limit $\Delta = 0$. Discuss the effect of dimerization as a function of the Luttinger parameter g (and Δ).