PHY-715: Solid State Physics, UMass Amherst, Problem Set #1

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Due: Friday Feb. 19, 2021.

I. DIMENSIONAL DEPENDENCE OF THE DEBYE LAW

Often, we are interested in the behavior of systems of reduced dimensionality, *e.g.* two-dimensional thin films, or one-dimensional wires. Determine how the Debye law for low-temperature scaling of the specific heat of vibrations changes with the dimension d of the system. (In this problem, we are only interested in the scaling of the specific heat with temperature T, so that you can ignore irrelevant constant prefactors).

II. GAPLESS MODES AND HEAT CAPACITY

Suppose there are some gapless excitations in a solid with dispersion relation $\omega \sim k^z$ (*e.g.* spin waves), with some dynamical exponent z. These excitations are completely decoupled from the vibrational degrees of freedom in the solid that give the familiar Debye contribution to the specific heat at low temperatures. Assuming that the additional excitations propagate in three dimensions (*i.e.*, are not confined to the surface), determine their additional contribution to the specific heat (up to some dimensionless integral that you do not have to compute), and show that the temperature-dependence can be used to extract the exponent z.

III. PAULI PARAMAGNETISM

The goal of this problem is to calculate the contribution of the electron spin to its magnetic susceptibility. Consider a gas of N non-interacting electrons in a volume V subject to a magnetic field $\vec{B}$, with single-particle Hamiltonian

$$H_1 = \frac{\vec{p}^2}{2m} - \mu_0 \vec{\sigma}.\vec{B},$$

where we have ignored orbital effects $\vec{p} \to \vec{p} - e\vec{A}$.

1. Compute the grand canonical potential Ω of this gas at a chemical potential μ.
2. Compute the number $N_\uparrow$, $N_\downarrow$ of electrons with spins pointing parallel and anti-parallel to the field as a function of the temperature T, magnetic field B and fugacity $z = \mathrm{e}^{\beta\mu}$.
3. Deduce the expression of the magnetization $m = M/N = \mu_0(N_\uparrow - N_\downarrow)/N$ at high temperatures as a function of T and B.
4. Analyze the low temperature behavior of the magnetization and compute the zero-field susceptibility $\chi = \frac{\partial M}{\partial B}\big|_{B=0}$ of this gas at zero temperature $T = 0$. Express your result in terms of the single-particle density of states at the Fermi energy.