PHY-715: Solid State Physics, UMass Amherst, Problem Set #4

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Due: Friday, April 23.

I. ELECTRICAL TRANSPORT IN $d = 1$

Consider a one-dimensional tight-binding chain near $T = 0$. Suppose that the right end of this chain is attached to a reservoir at chemical potential μ_R and the left end is attached to one at chemical potential μ_L, and assume $\mu_L > \mu_R$.

1. Decompose the electric current into contributions from right- and left-moving electrons $J_{\text{tot}} = j_R - j_L$ and carefully argue that:

$$j_{R/L} = -e \int_{k>0} \frac{dk}{\pi} v_k n_F[\beta(E(k) - \mu_{L/R})]. \tag{1}$$

where $v_k = \frac{1}{\hbar}\frac{dE(k)}{dk}$ is the group velocity and n_F the usual Fermi function.

2. Calculate the conductance G of the wire, defined via $J_{\text{tot}} = GV$ where $J_{\text{tot}} = j_R - j_L$ and $-eV = \mu_L - \mu_R$, and show it is equal to $G = 2e^2/h$ at $T = 0$. The ratio of fundamental constants e^2/h is the "quantum of conductance".

II. THERMAL TRANSPORT IN $d = 1$

Return to the one-dimensional problem studied above, but now suppose the reservoirs have the *same* chemical potential $\mu = 0$, but different (low) temperatures, T_L, T_R.

1. Compute the contribution to the *energy current* $j^Q_{R/L}$ of all particles moving to the right (left), similarly to eq. (1) but for energy transport. (With the total energy current from left to right $J^Q = j^Q_R - j^Q_L$.)

2. Define the *thermal* conductance K via $J^Q = K\Delta T$ where $J^Q = j^Q_R - j^Q_L$ and $T_{L,R} = T \pm \Delta T/2$, and $\Delta T \ll T$. Express K as an integral over energy involving $\frac{dn_F}{dE}$, and evaluate this integral at low temperature to confirm that the Weidemann-Franz ratio for clean 1D systems (defined here in terms of conductances rather than conductivities) is

$$\frac{K}{TG} = \frac{\pi^2 k_B^2}{3e^2}.$$

You'll need the integral $\int_{-\infty}^{\infty} dx \frac{x^2 e^x}{(1+e^x)^2} = \frac{\pi^2}{3}$.

III. DRUDE MODEL REVISITED

In the semi-classical approximation, the motion of an electron of charge $-e$ in an external electric field $\vec{E}$ is determined by the Drude model

$$\hat{m}_\star \frac{d\vec{v}}{dt} = -e\vec{E} - \frac{\hat{m}_\star \vec{v}}{\tau},$$

with τ the scattering time, and $\hat{m}_\star$ is the effective mass 3×3 matrix. Assume that the electrons are subjected to an oscillating electric field of the form $\vec{E} = \vec{E}_\omega e^{-i\omega t}$. The electric current is defined as $\vec{j} = -ne\vec{v}$ where n is the density of electrons. Show that the electric current takes the form $\vec{j} = \vec{j}_\omega e^{-i\omega t}$ where $\vec{j}_\omega$ is given by Ohm's law, $\vec{j}_\omega = \hat{\sigma}(\omega)\vec{E}_\omega$, where you will express the conductivity matrix $\hat{\sigma}(\omega)$ in terms of n, e, τ, ω and $\hat{m}_\star$.