

# Lectures on diffusion in complex and random media

David S. Dean

Laboratoire d'Ondes et Matière d'Aquitaine, Université de Bordeaux  
UMass Summer School on Soft Solids and Complex Fluids

May 2016

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.1	Subject of the Lectures . . . . .	2
1.2	A few remarks about notation . . . . .	2
<b>2</b>	<b>Crash course on stochastic calculus</b>	<b>4</b>
2.1	Discrete time continuous space stochastic processes . . . . .	4
2.2	The Ito Stochastic Calculus . . . . .	6
2.3	Examples of Stochastic Differential Equations . . . . .	7
2.4	The Generator and the Forward Fokker-Planck Equation . . . . .	10
2.5	Generalization to higher dimensions . . . . .	10
2.6	A brief word on other forms of Stochastic Calculus. . . . .	11
2.7	Links with physical descriptions of diffusion . . . . .	11
2.8	First passage times . . . . .	14
<b>3</b>	<b>Diffusion in media with variable diffusivity</b>	<b>19</b>
3.1	Links with electrical and porous media . . . . .	20
3.2	Diffusion with variable diffusivity in one dimension . . . . .	22
3.3	Wiener bounds . . . . .	22
3.4	Exact result in two dimensions . . . . .	23
3.5	Links between diffusion with varying diffusivity and diffusion in a potential	24
3.6	Dynamical transition in squared Gaussian potentials . . . . .	26
3.7	The Maxwell formula for diffusion in systems with spherical inclusions . . .	27
3.8	Diffusion in a system of hard spherical obstacles . . . . .	30
<b>4</b>	<b>Perturbation methods for systems with random drift</b>	<b>31</b>
4.1	Non-averaged perturbation theory . . . . .	32
4.2	Disorder averaged perturbation theory . . . . .	33