Introduction to pattern formation

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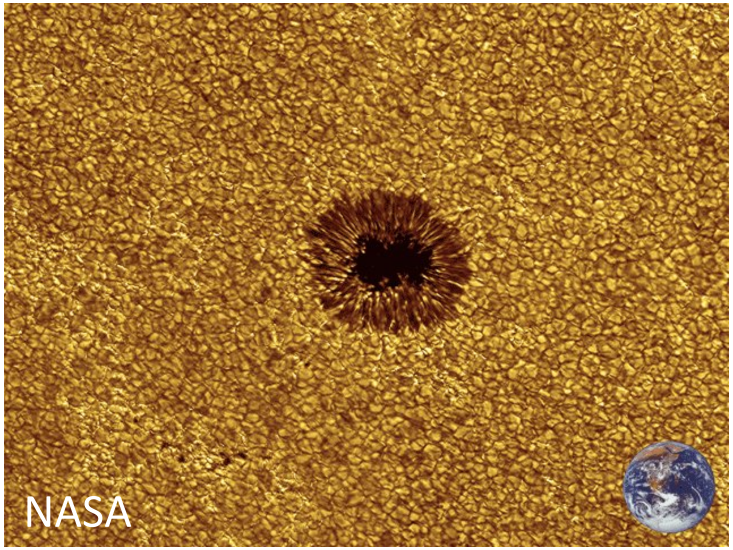
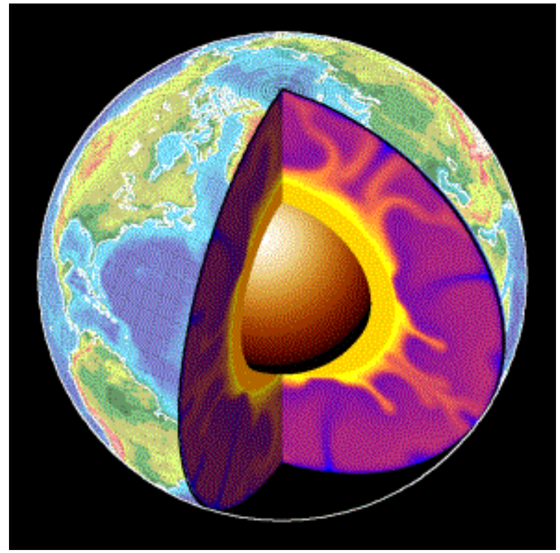
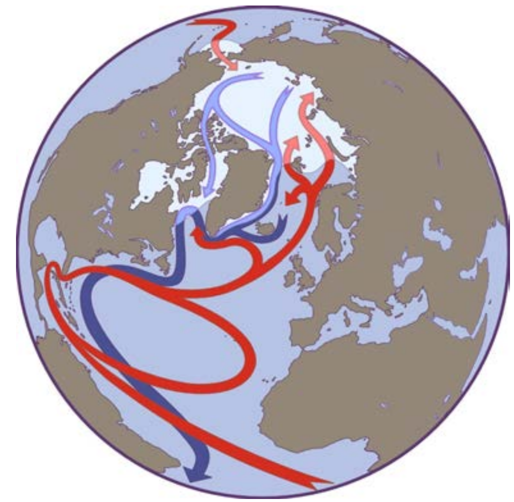
Physics, Complex Systems, & Math

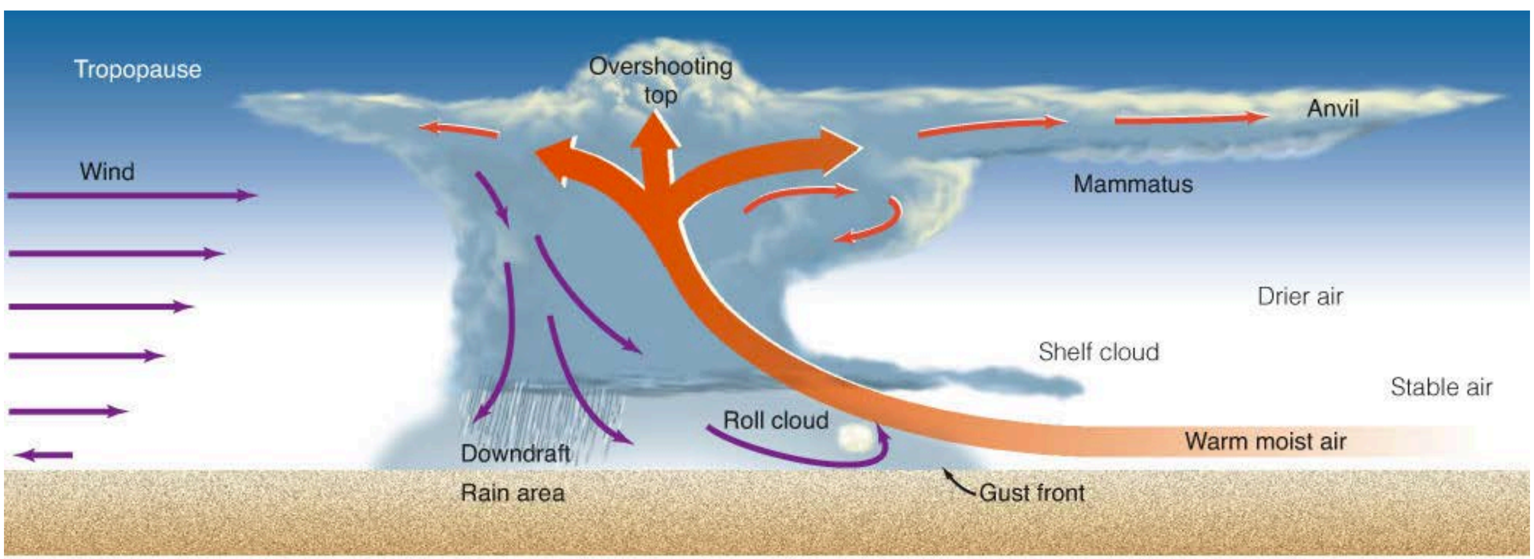
Driven systems

Convection

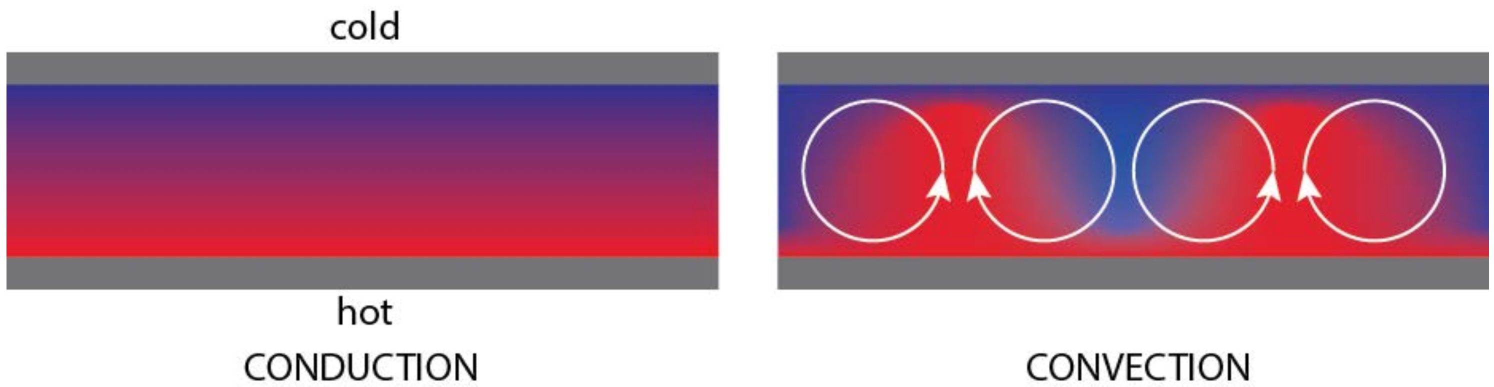

NASA



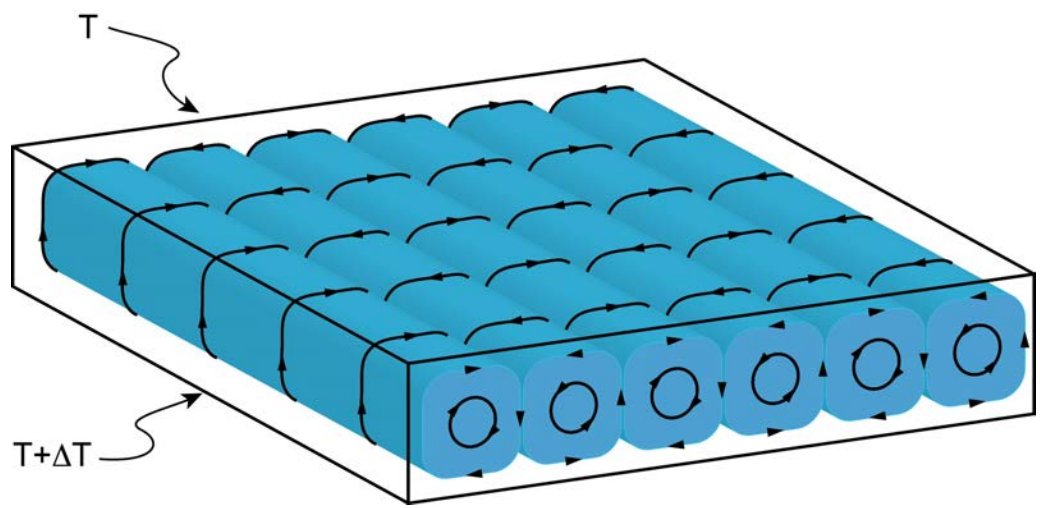



Tropopause
Overshooting top
Anvil
Wind
Mammatus
Drier air
Shelf cloud
Stable air
Roll cloud
Warm moist air
Downdraft
Rain area
Gust front

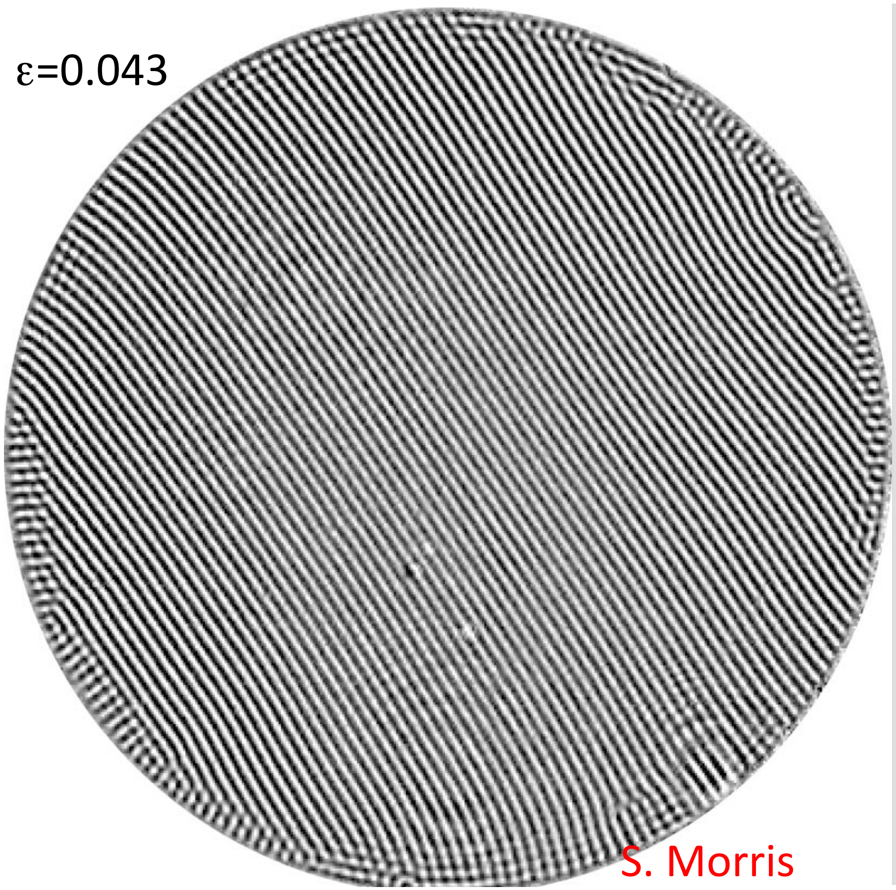
Rayleigh-Benard convection

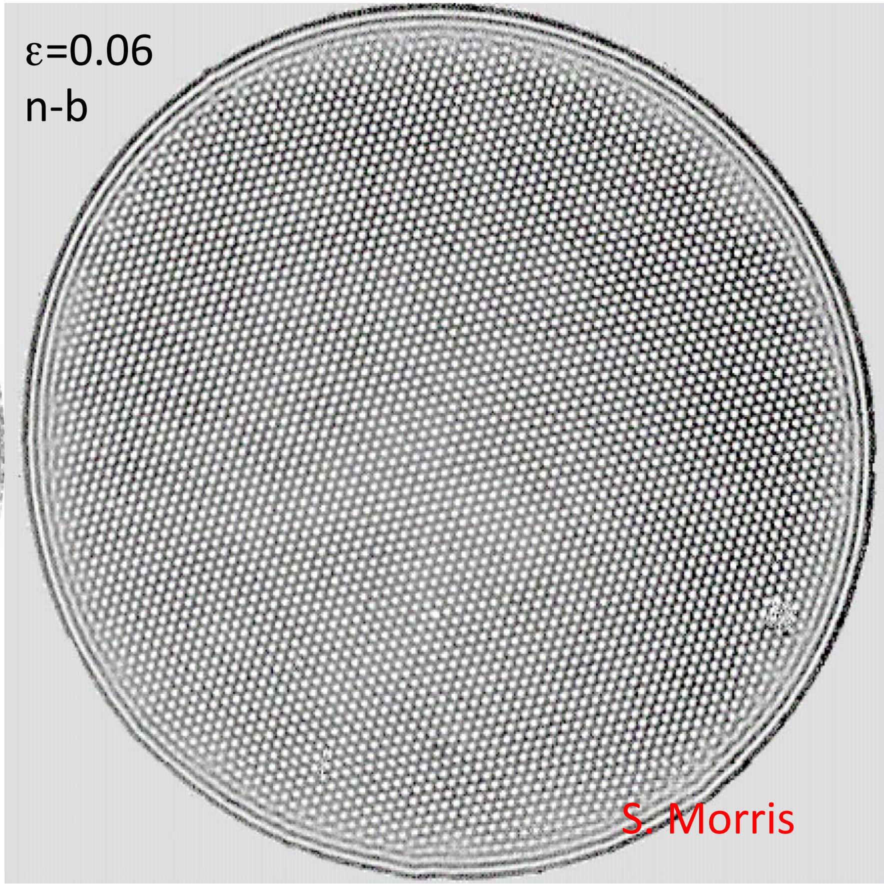


Control parameter $\varepsilon=(\Delta T-\Delta T_c)/\Delta T_c$

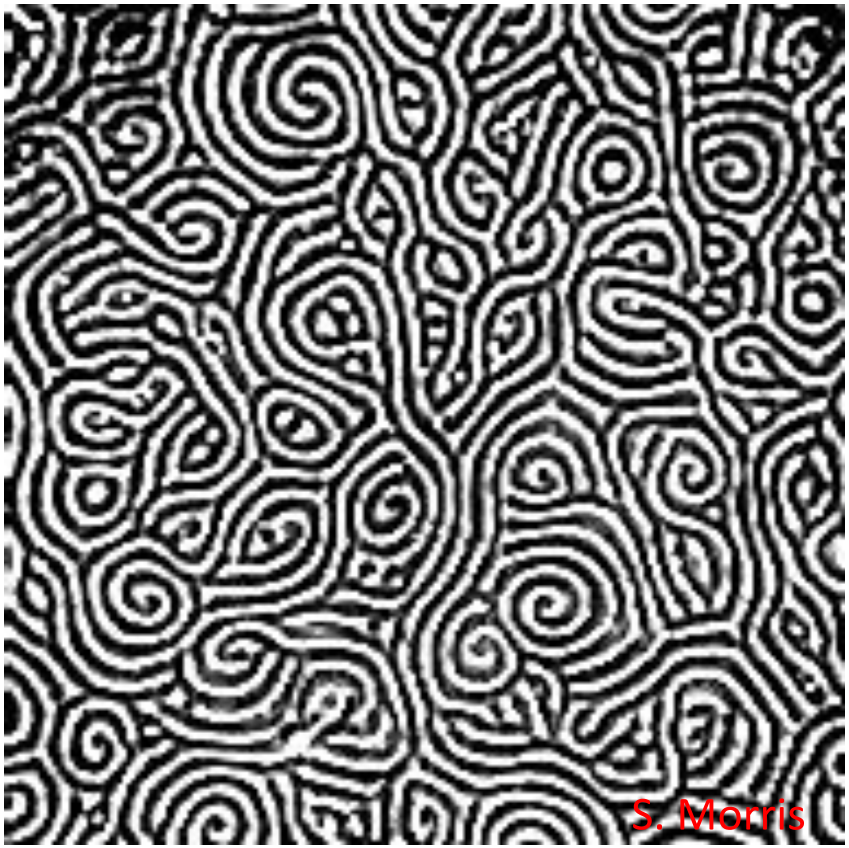


Rayleigh-Benard convection

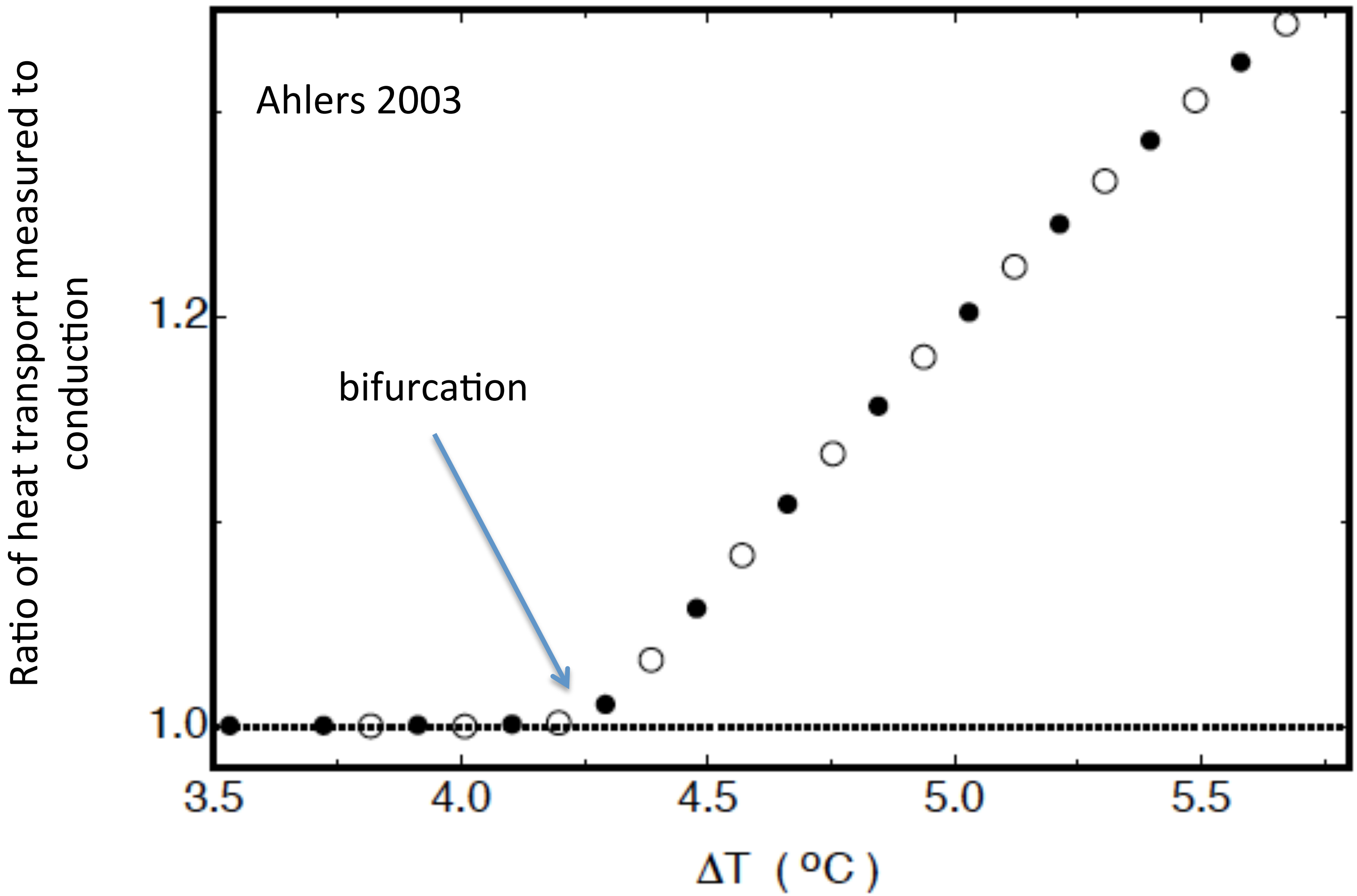




Rayleigh-Benard convection

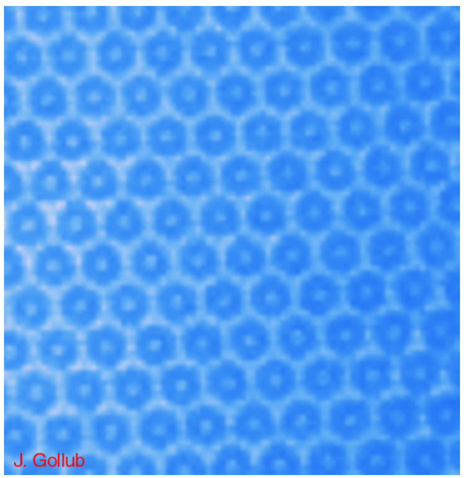


Rayleigh-Benard convection

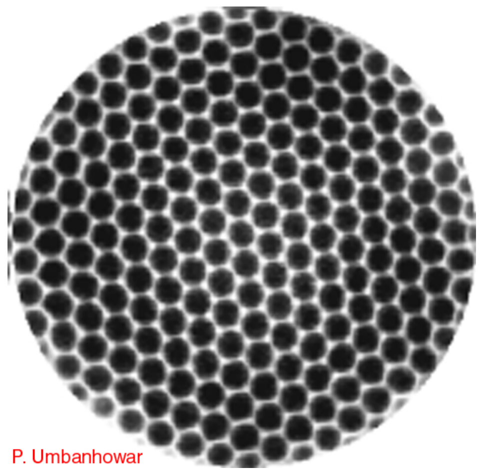


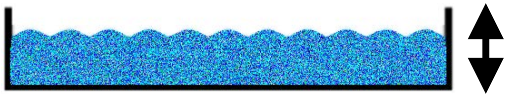
Universality: hexagons

Faraday waves

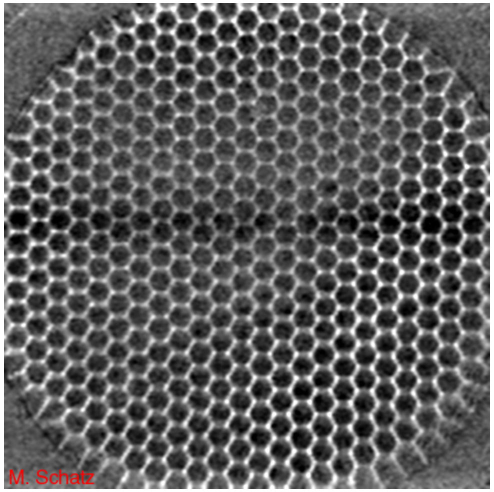


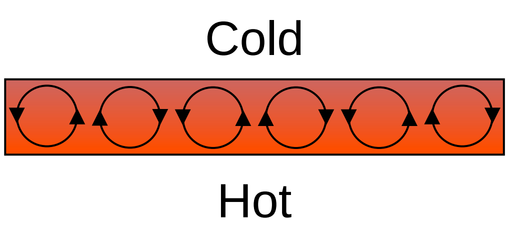
Vibrated sand



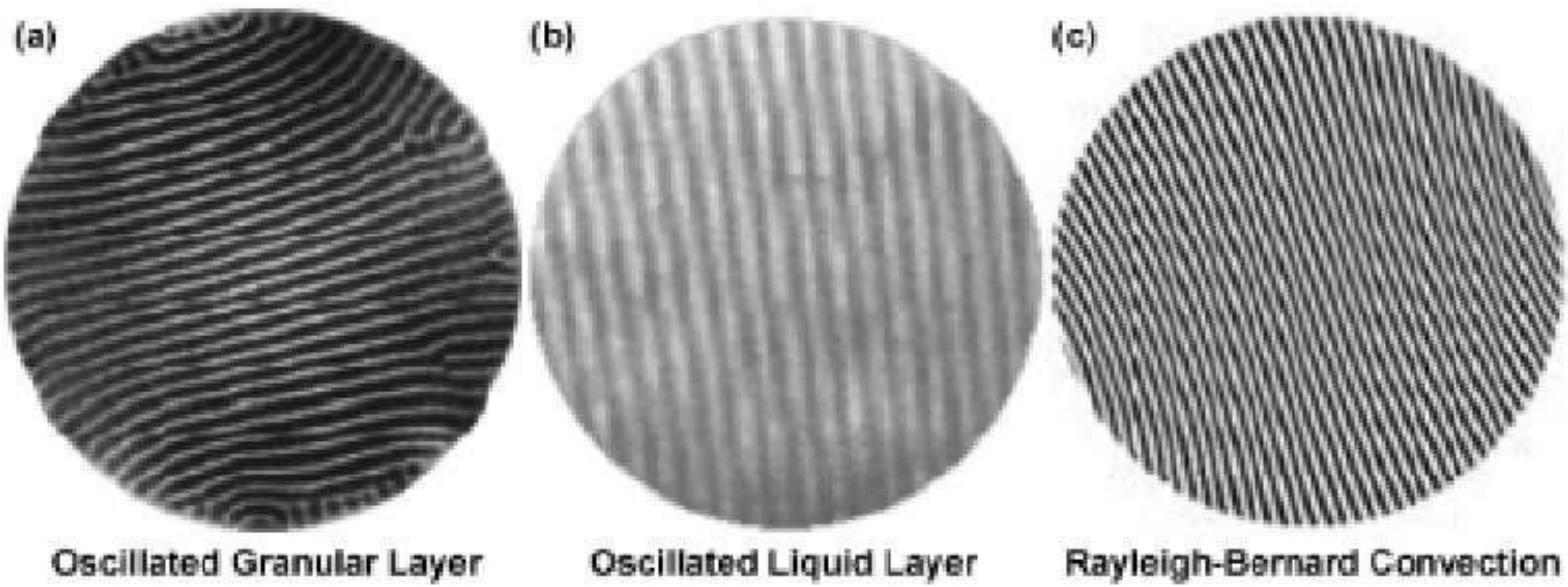


RBC



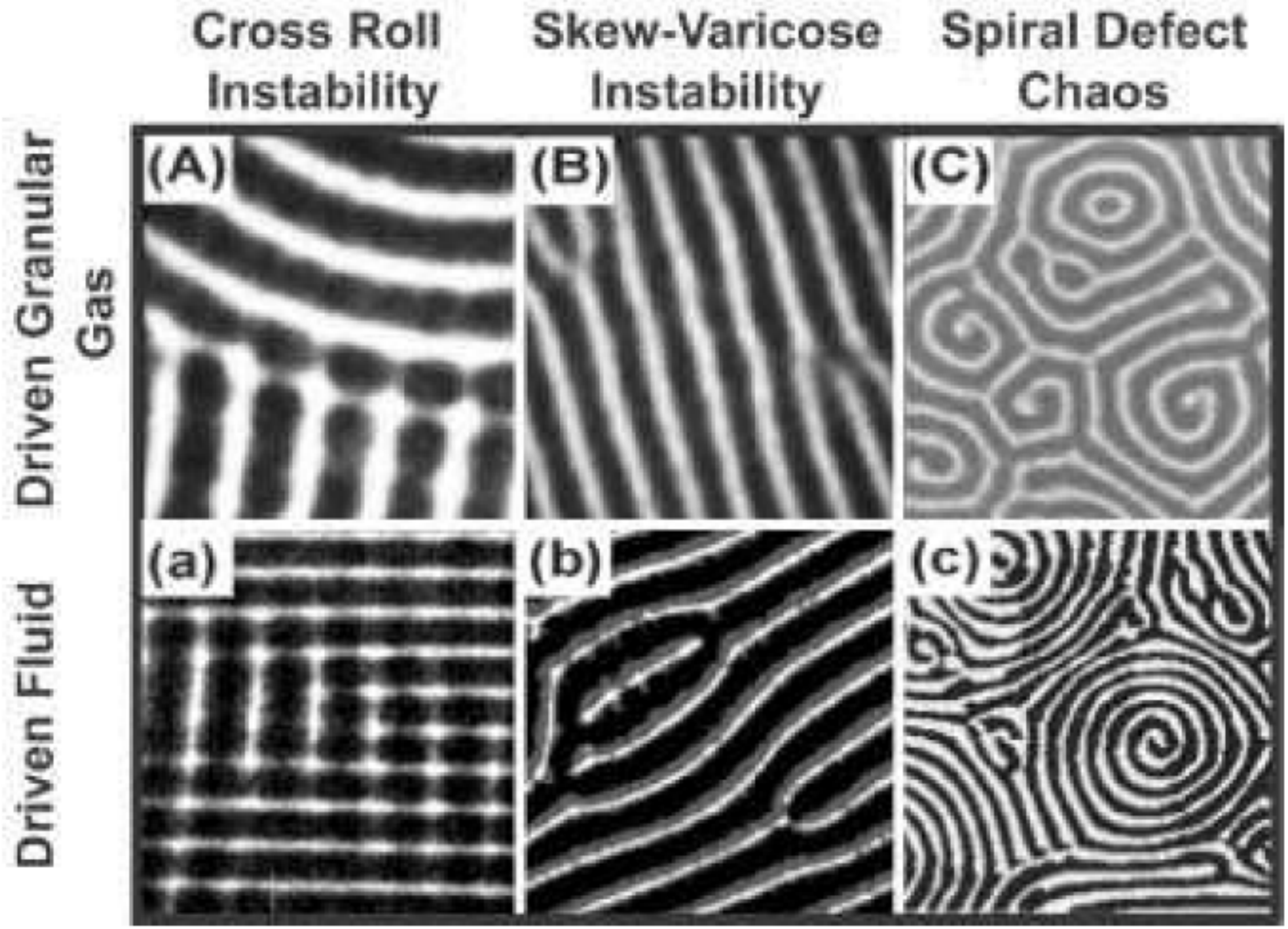


Universality: stripes



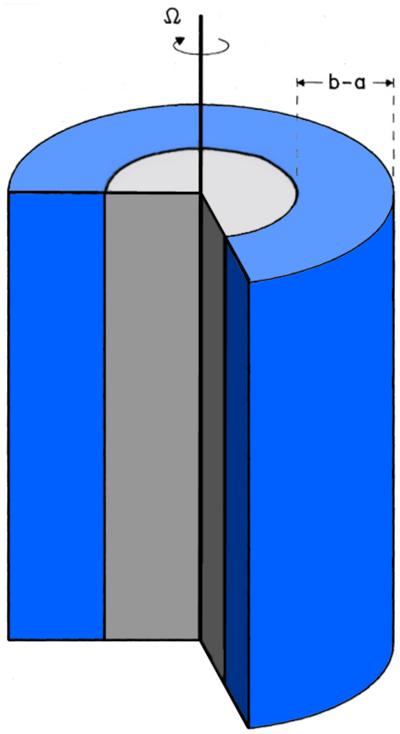
from Swinney & Rericha, 2004

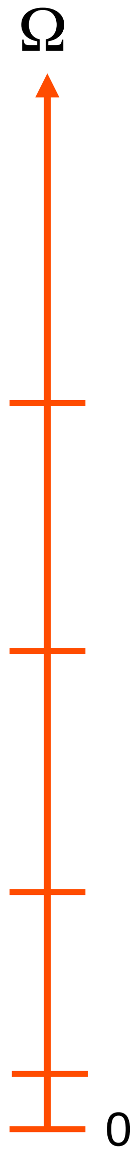
Universality: defects



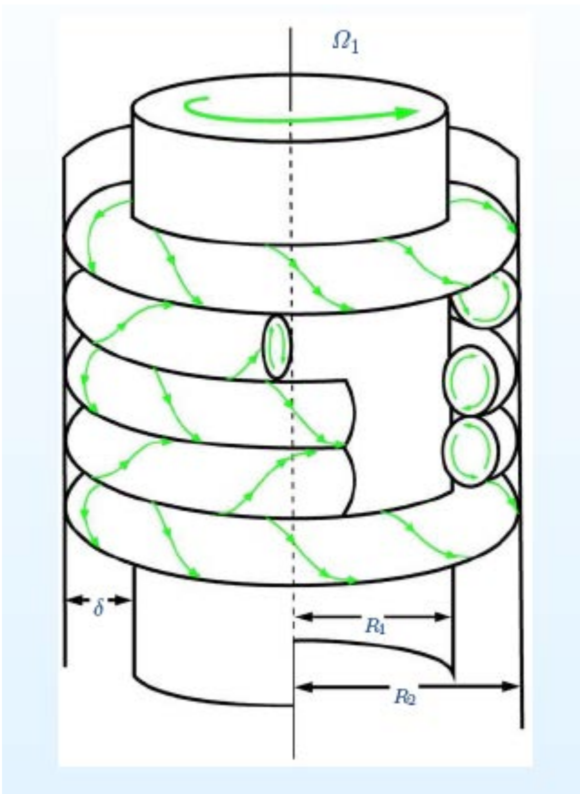
from Swinney & Rericha, 2004

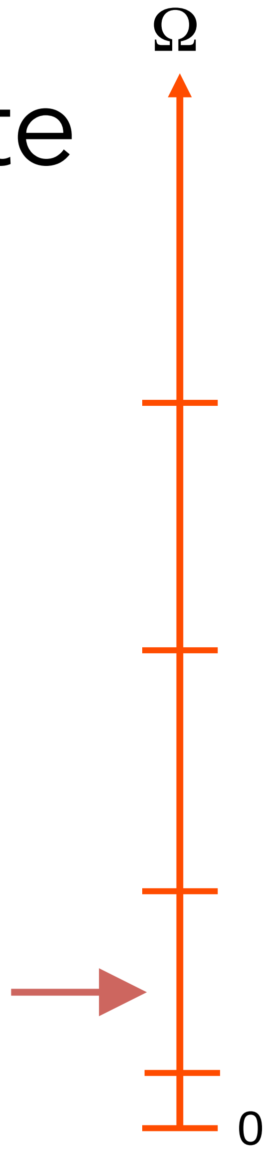
Taylor-Couette





Taylor-Couette





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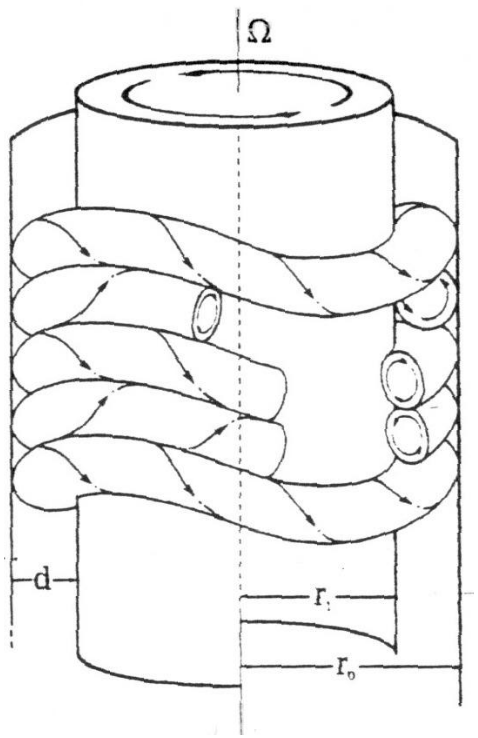

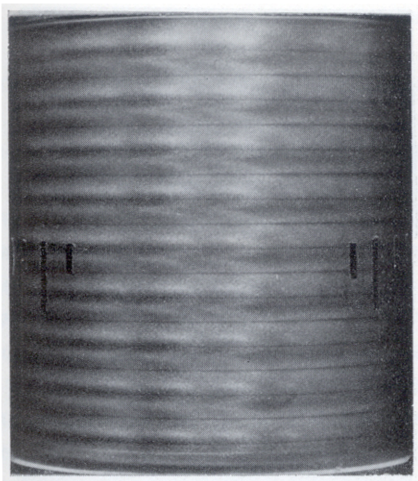

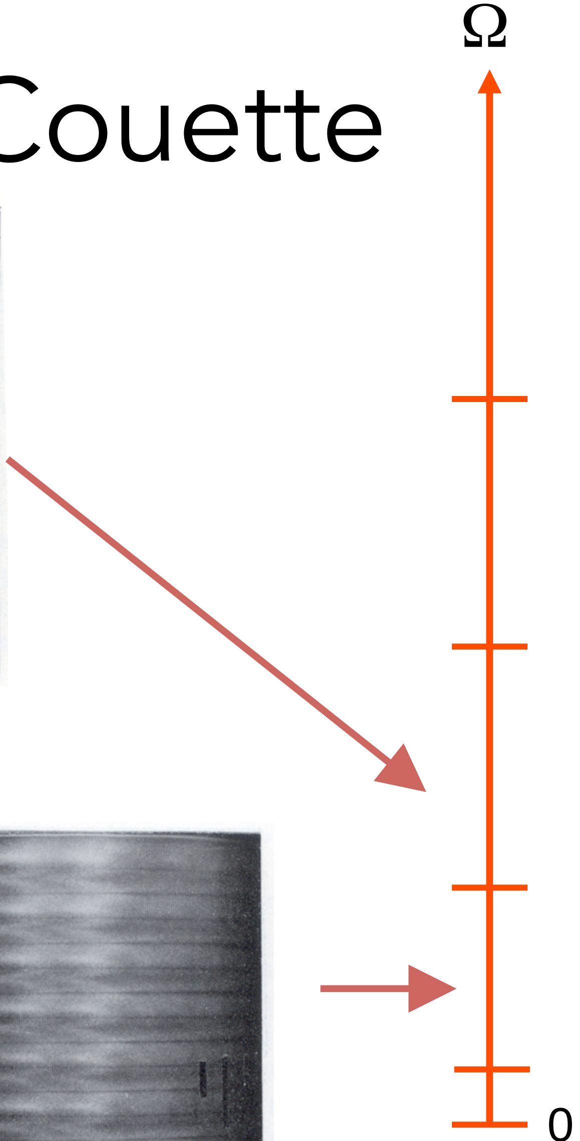

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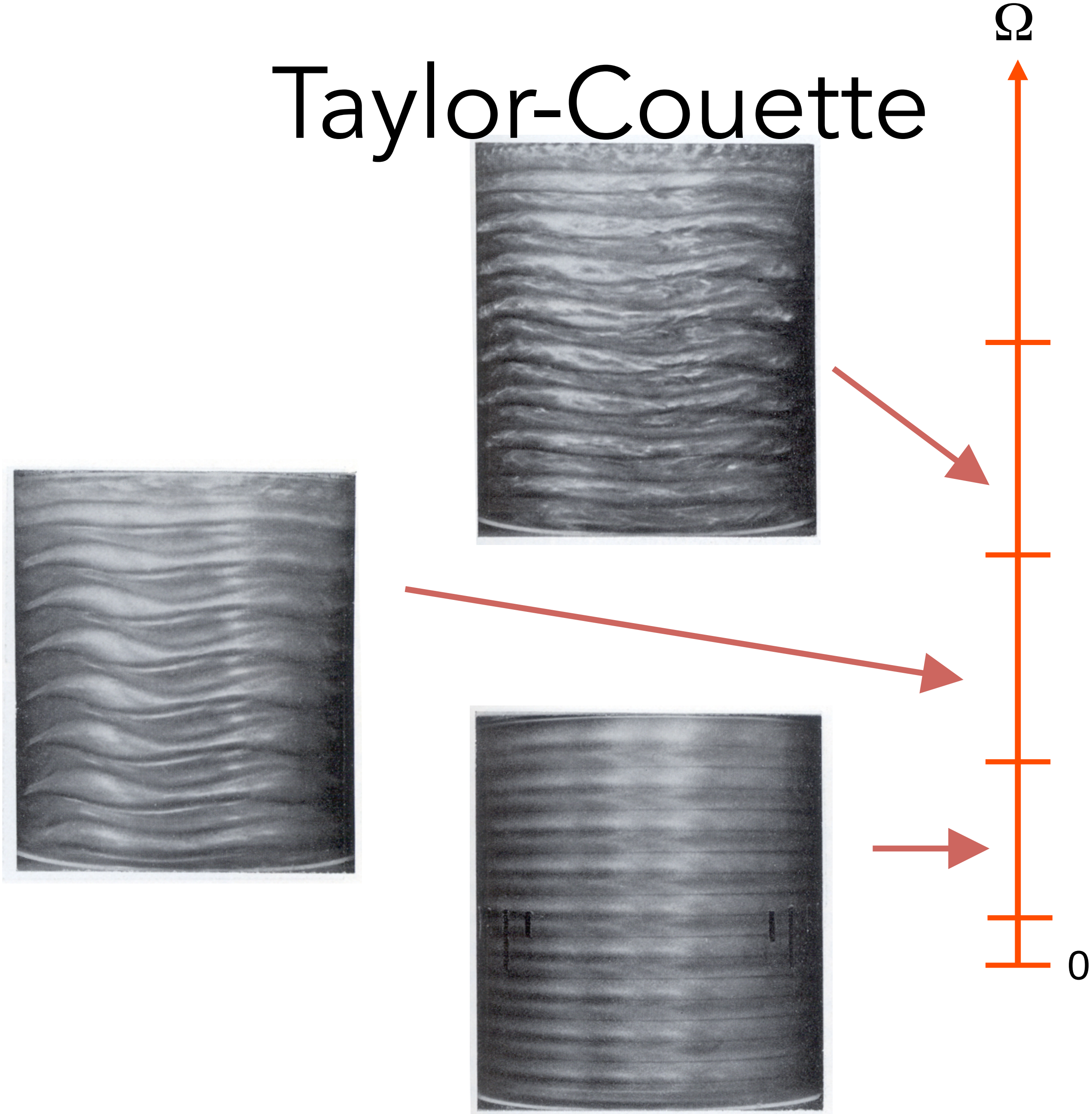

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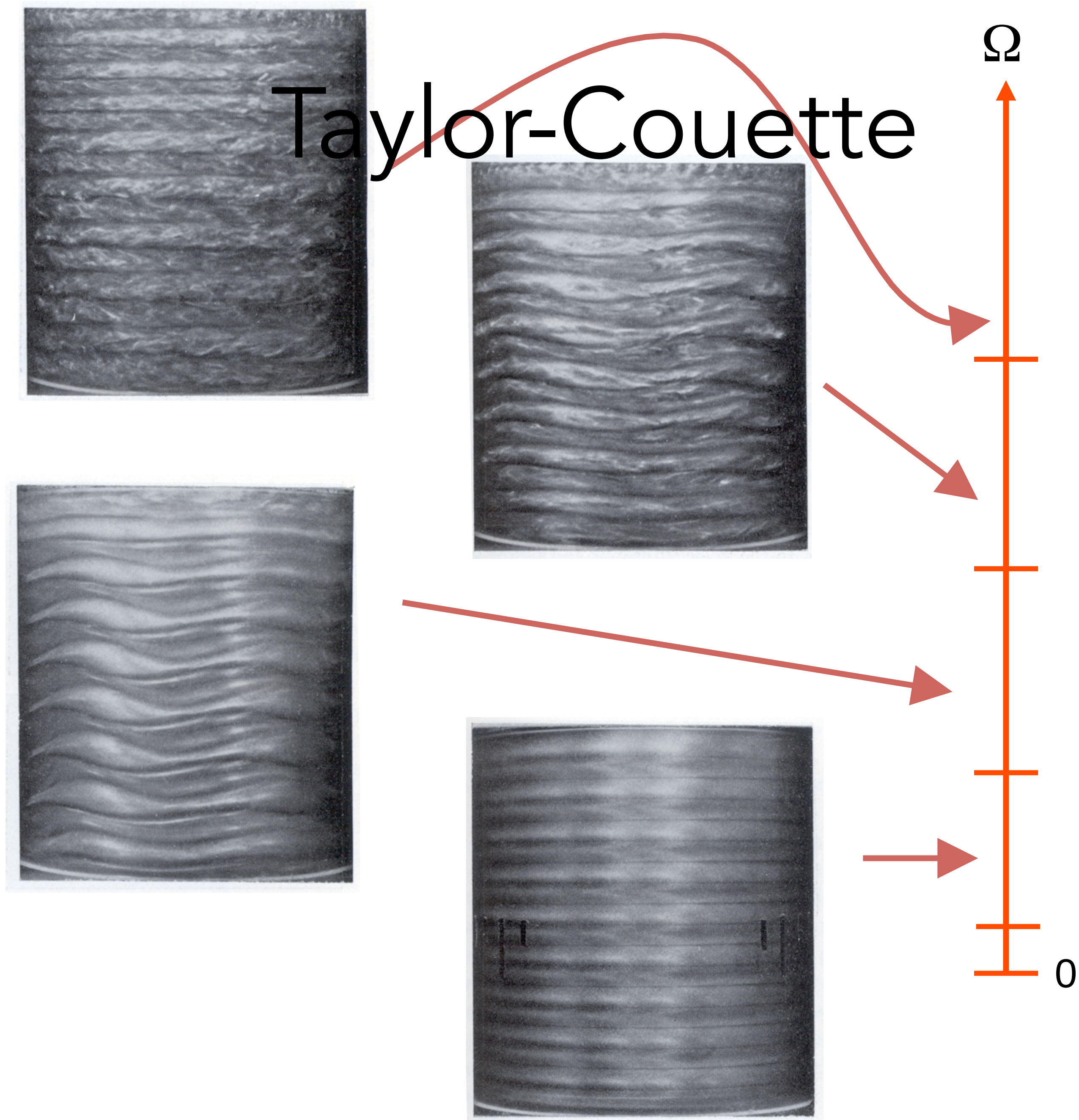

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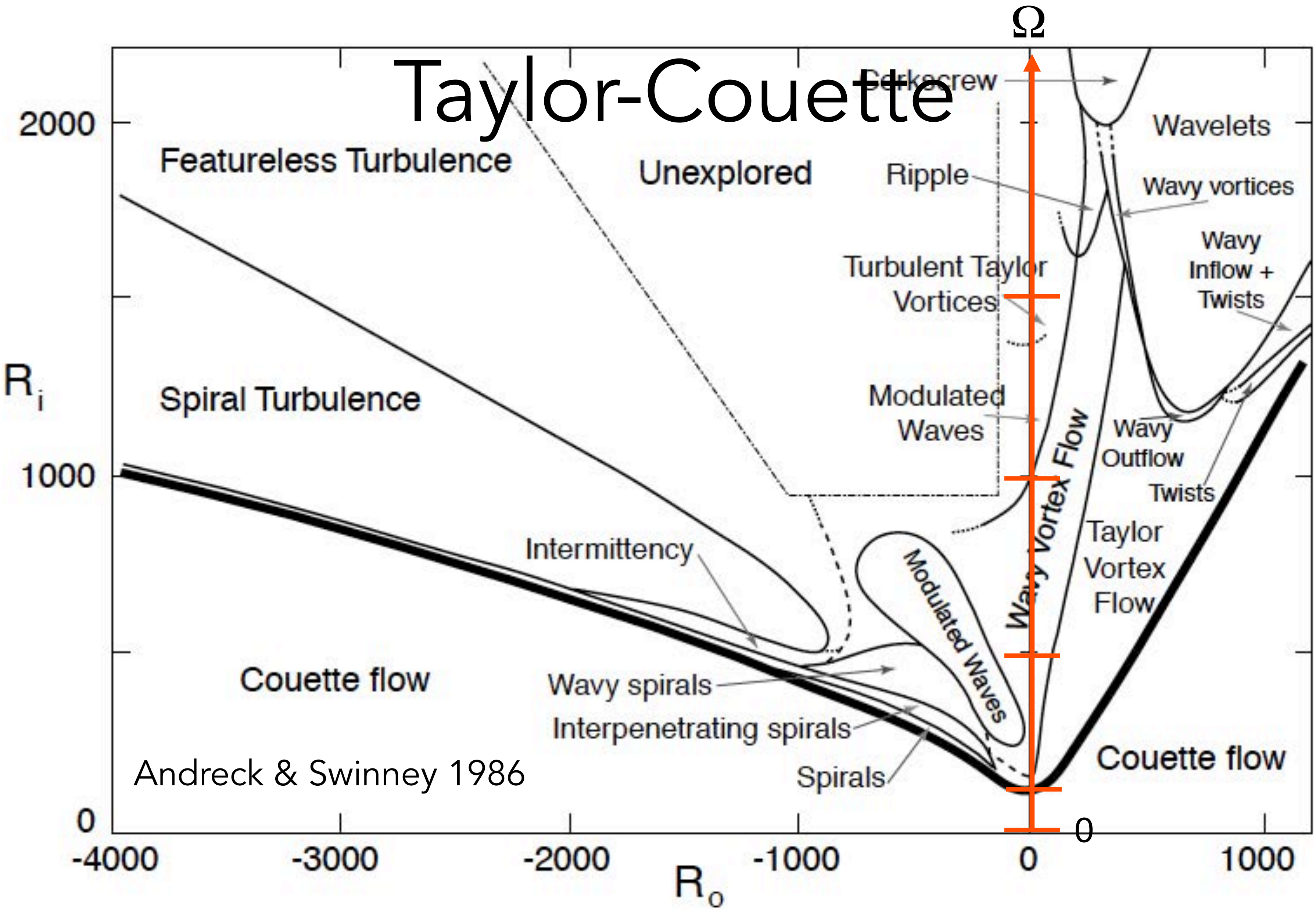
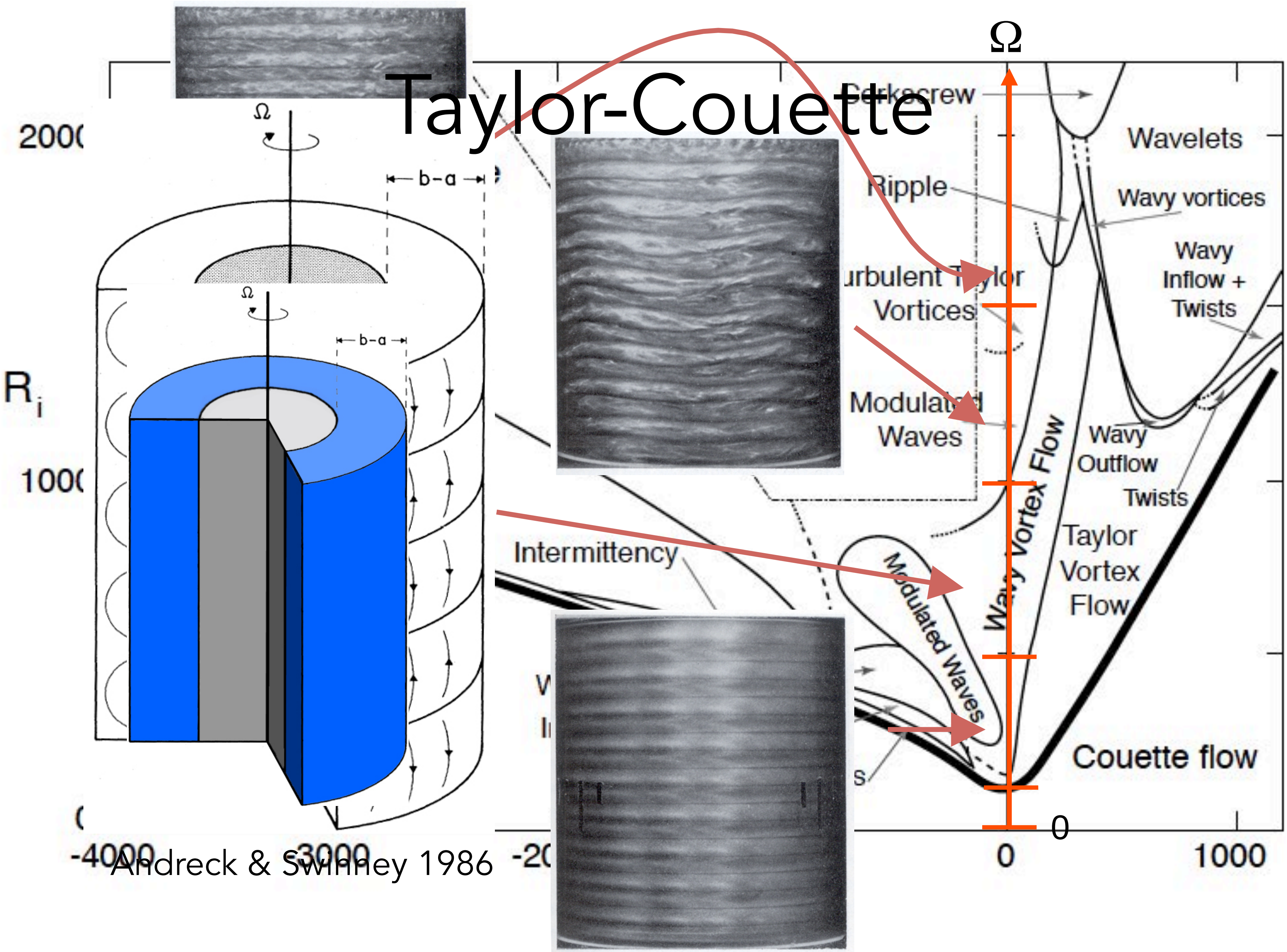


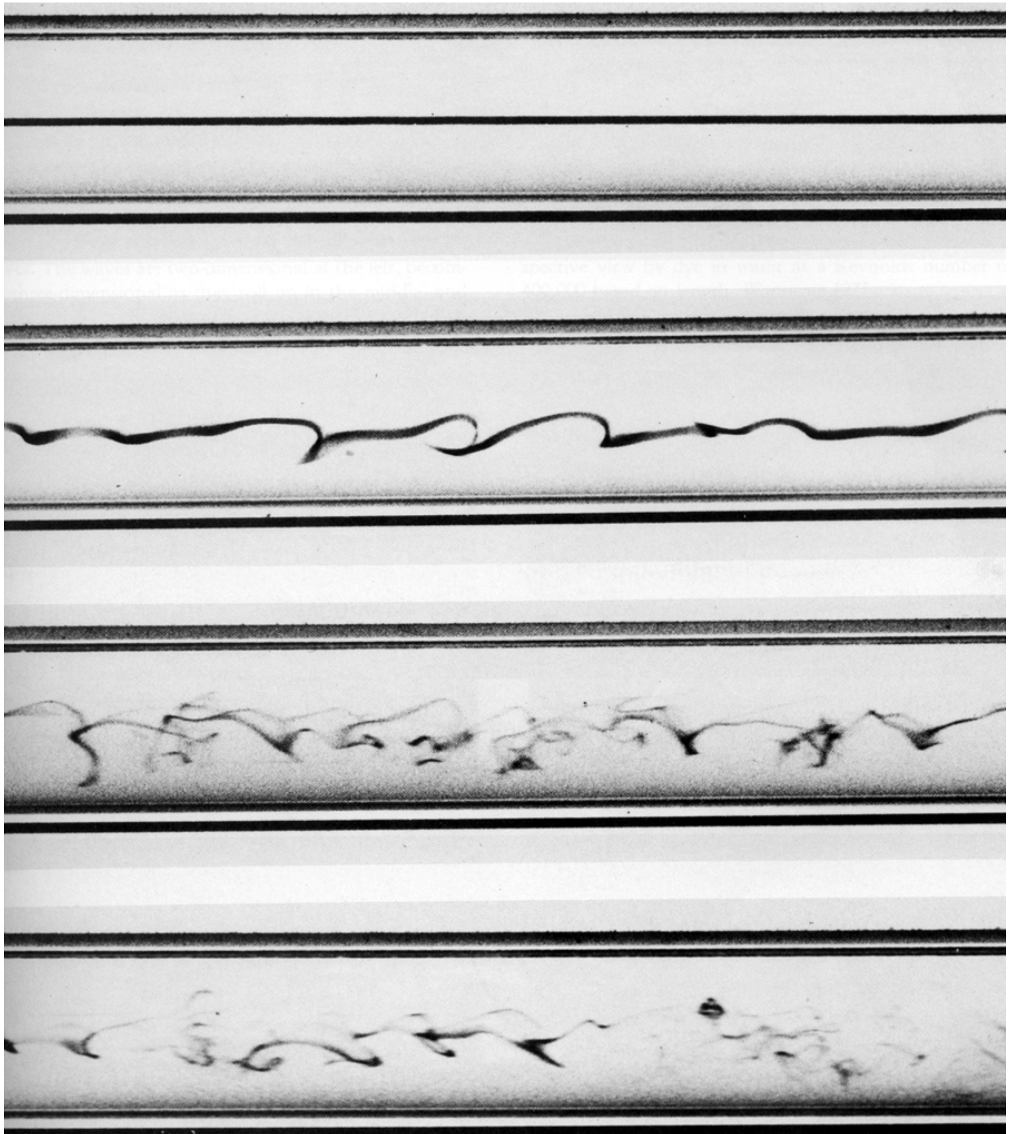
Andreck & Swinney 1986

Taylor-Couette

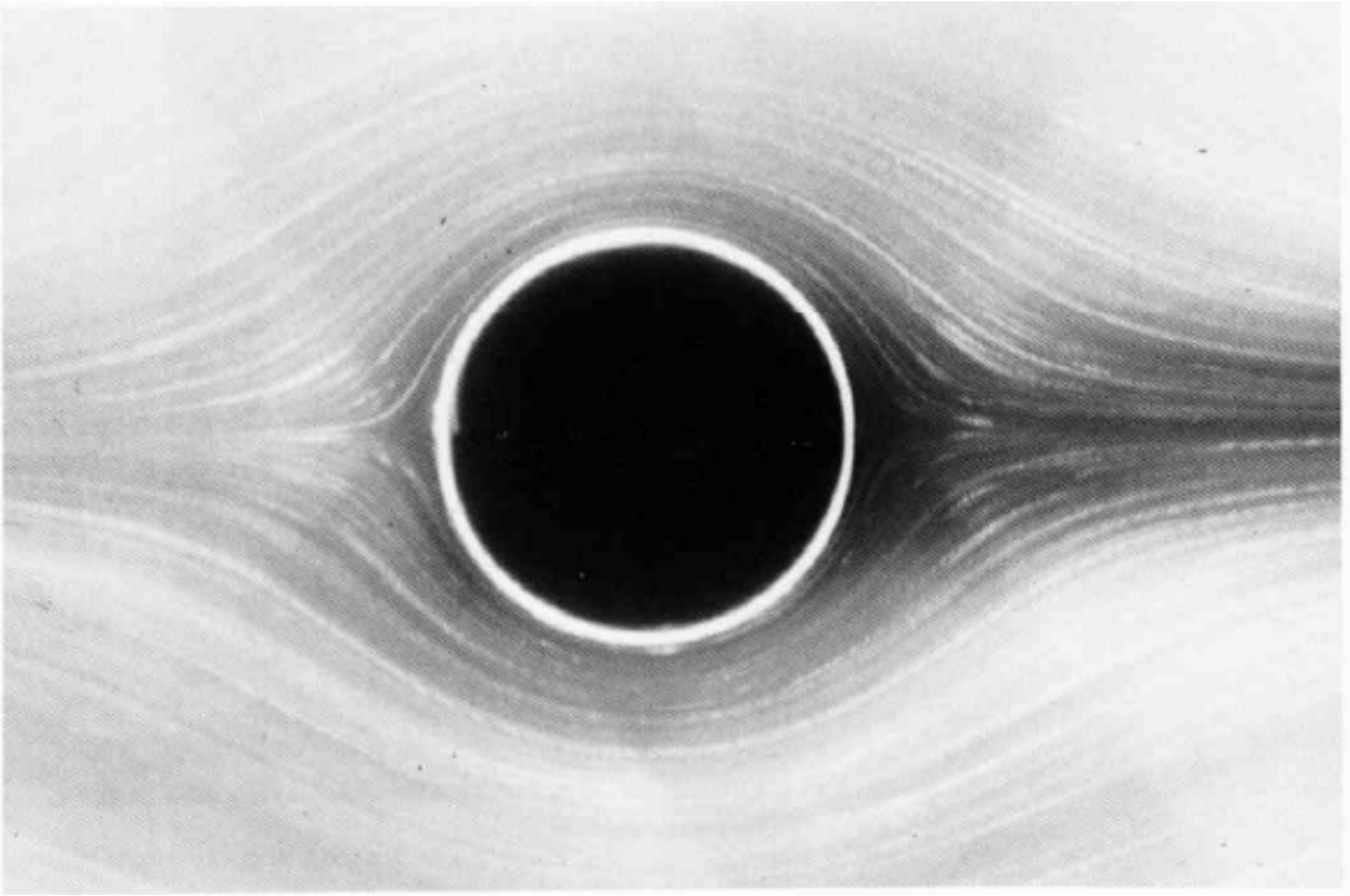


Andreck & Swinney 1986

Turbulence

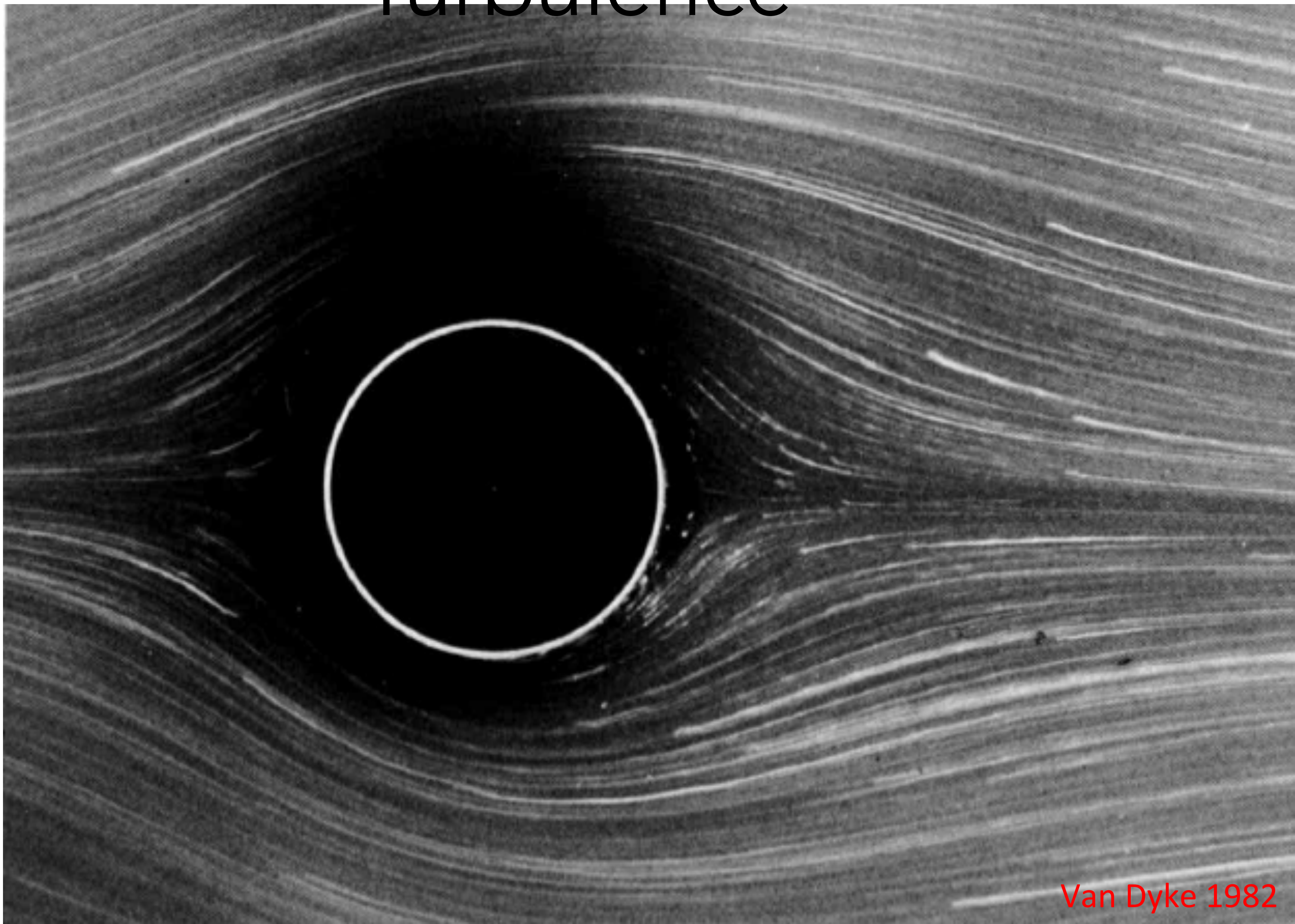


Turbulence

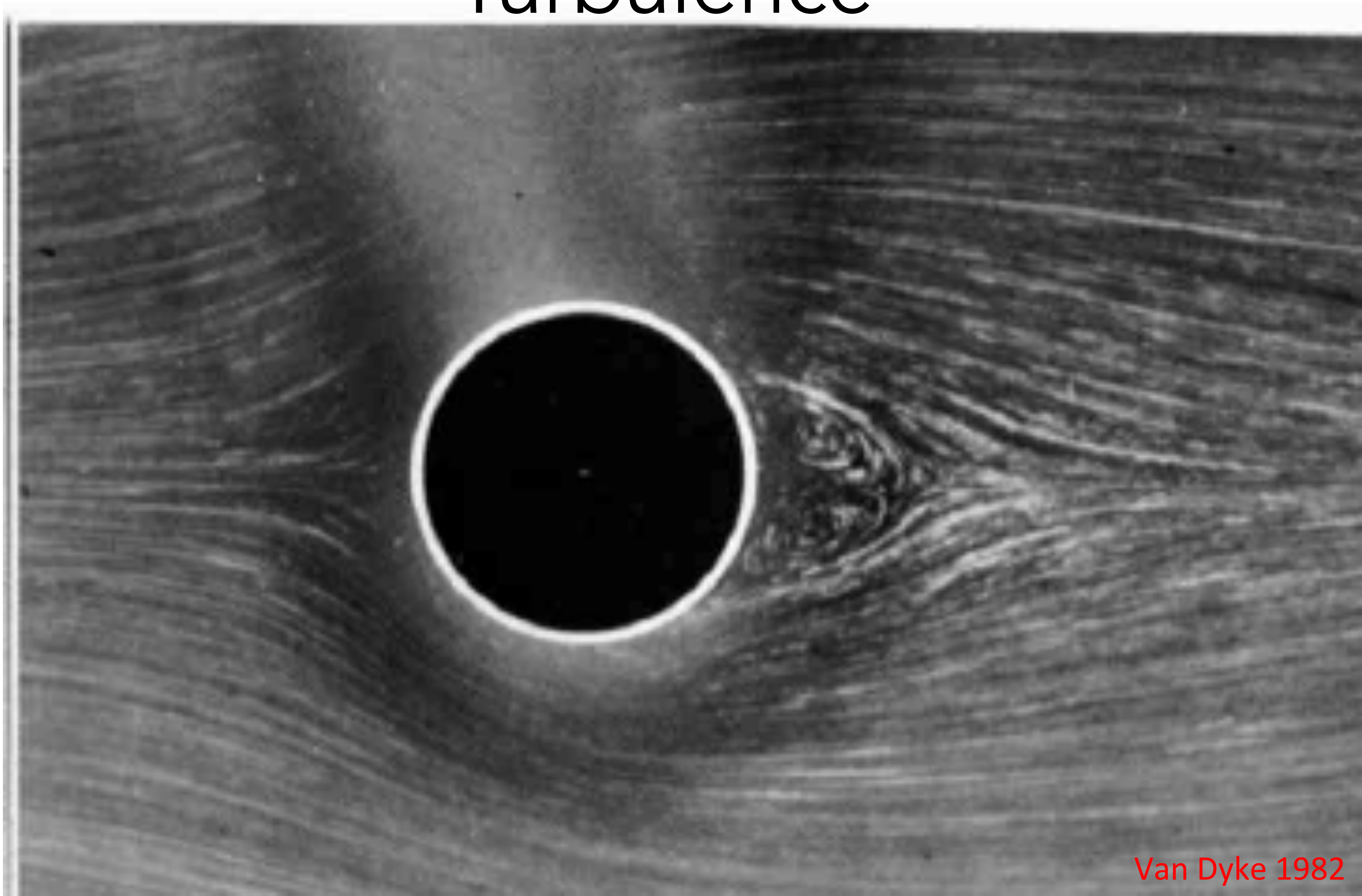


Van Dyke 1982

Turbulence

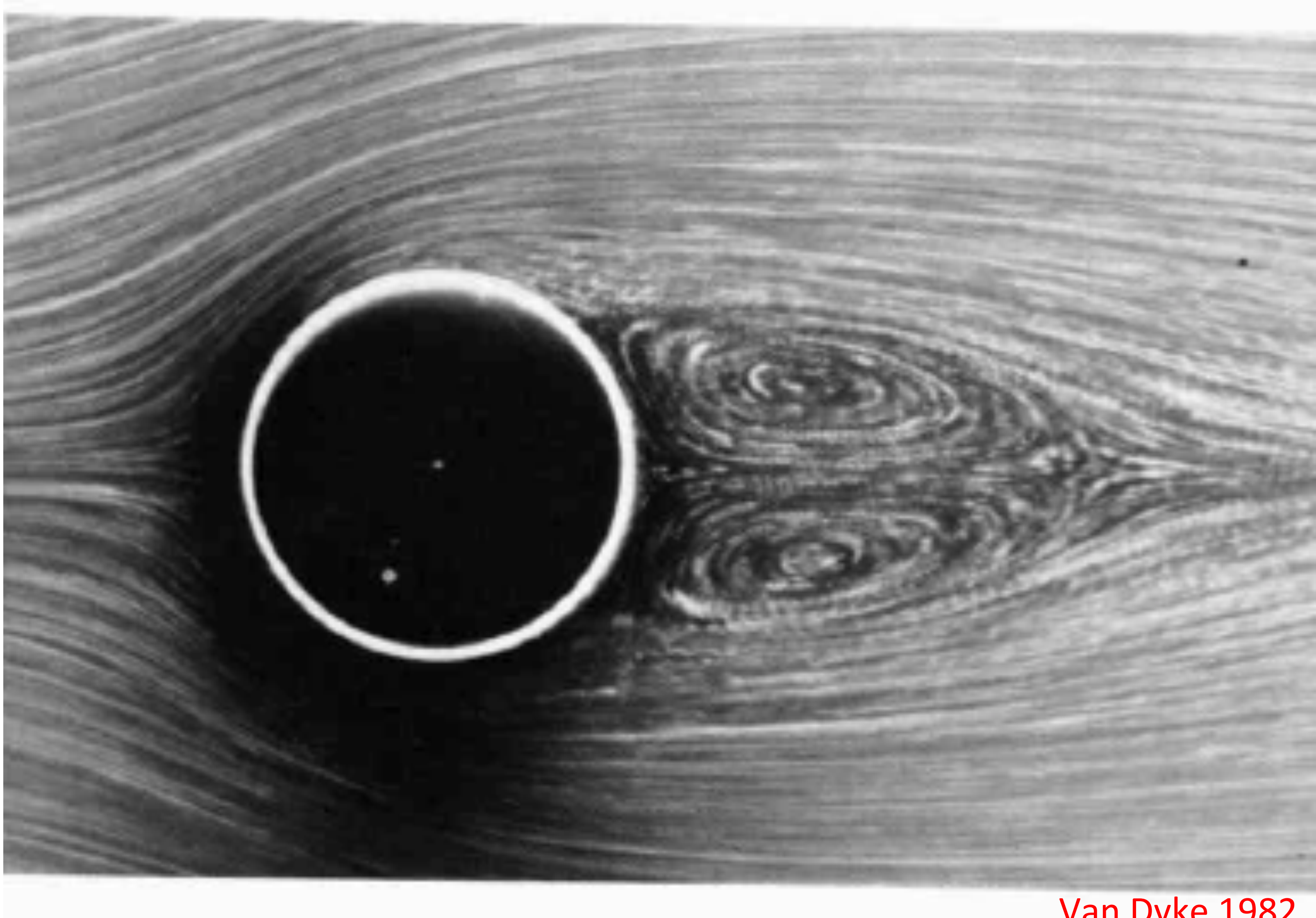


Turbulence



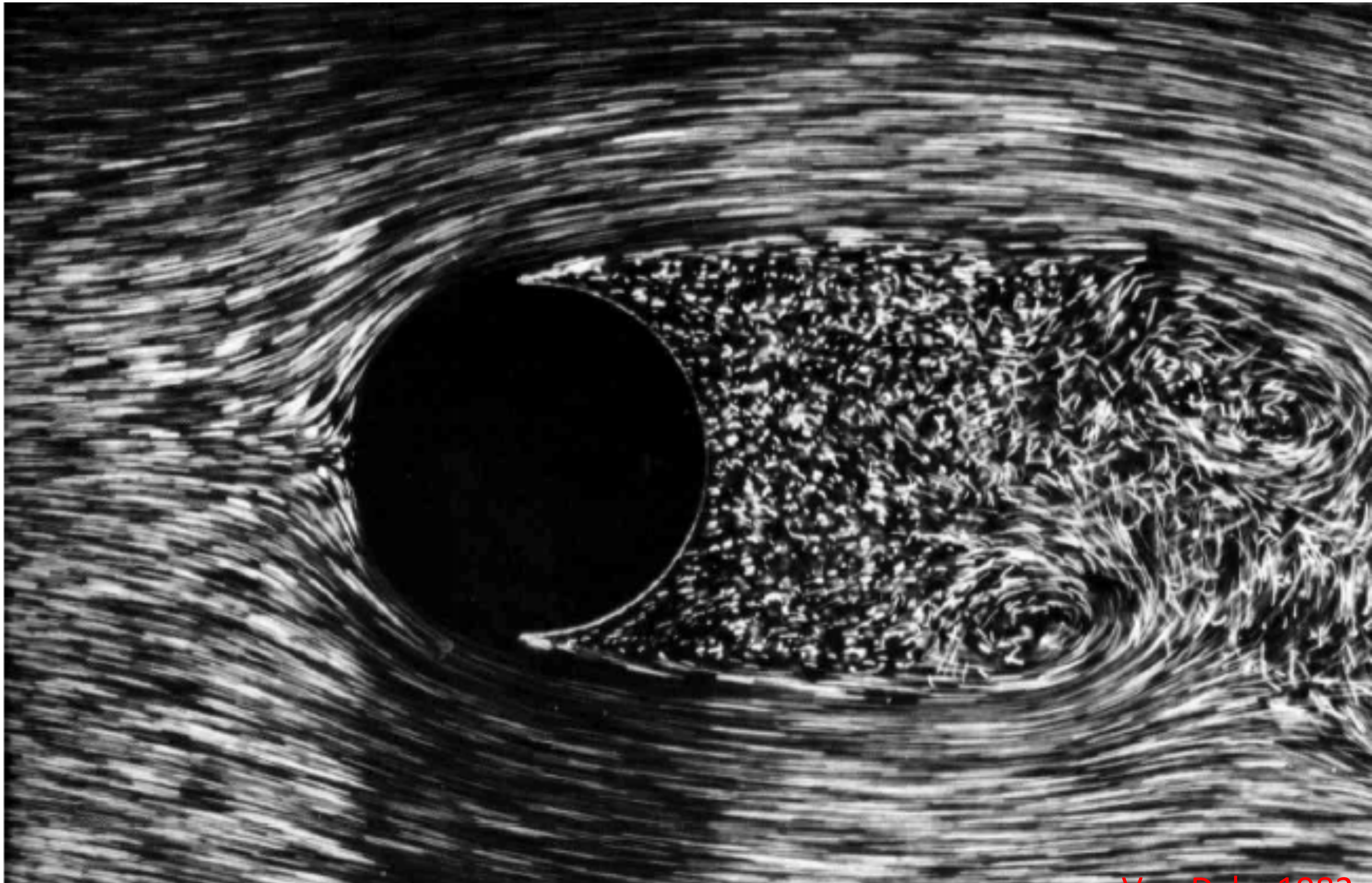
Van Dyke 1982

Turbulence



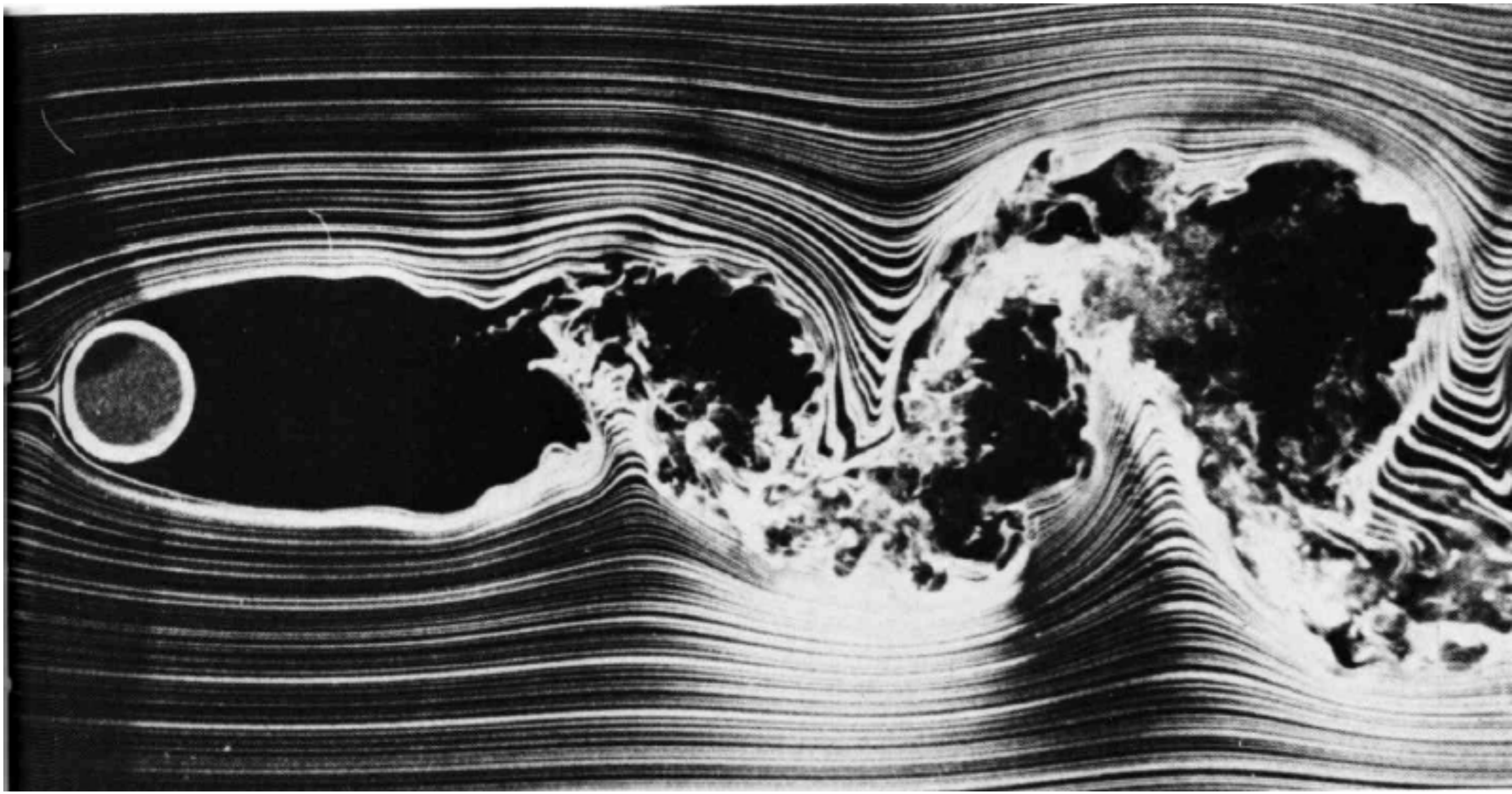
Van Dyke 1982

Turbulence



Van Dyke 1982

Turbulence

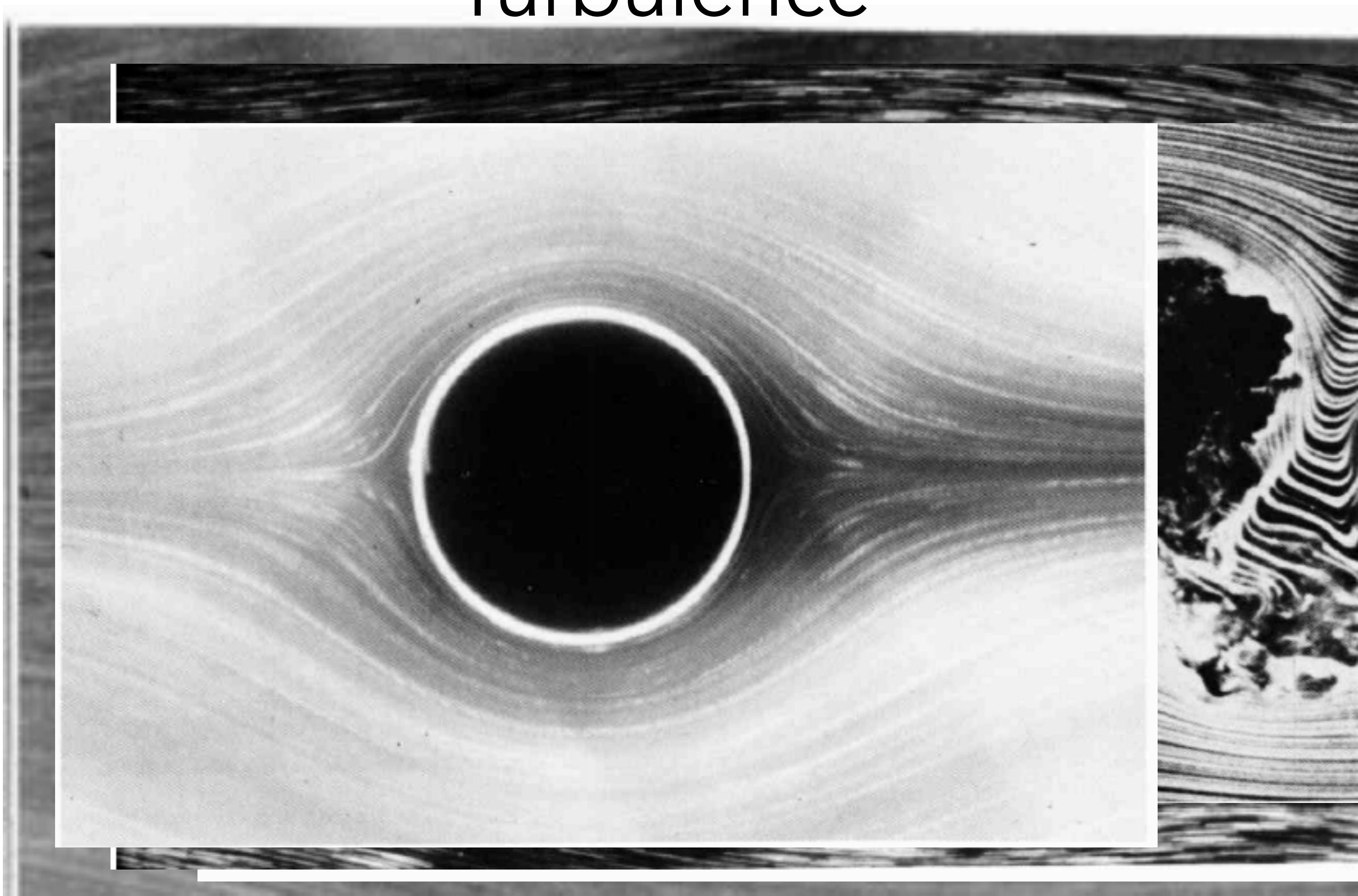


Van Dyke 1982

Common features

- External boundary conditions fixed in non-equilibrium configurations.
- Boundary conditions act as control parameter
- Past parameter threshold → spontaneous broken symmetry to "structured" state
- Structured states exhibit structure with specific length scale
- Secondary transitions to other states
- Universality

Turbulence



Pattern formation questions

Fundamental question:
Where do patterns come from?

1. What is the origin of the instability?
2. What sets the wavelength?
3. What causes the instability to saturate?
4. How is the pattern selected?
5. What are the transitions?

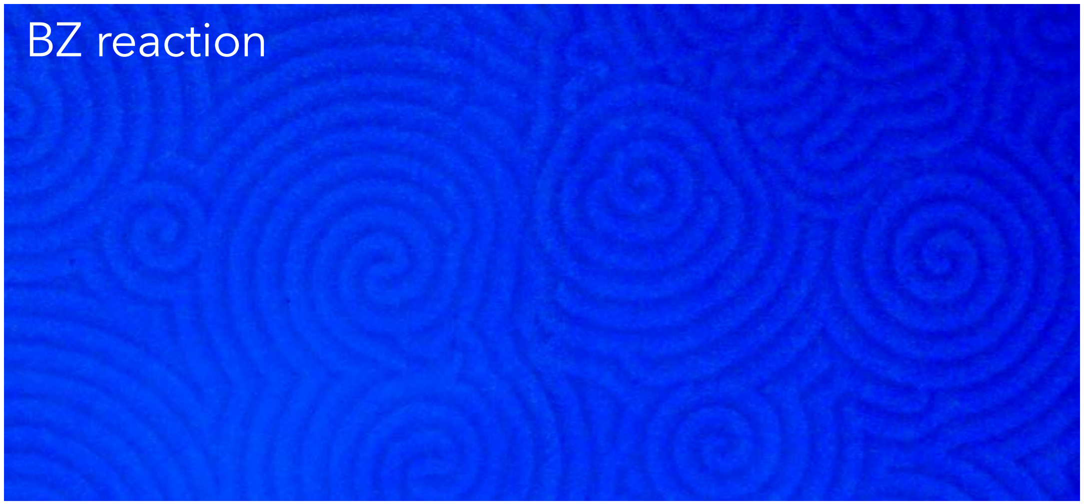
Pattern formation

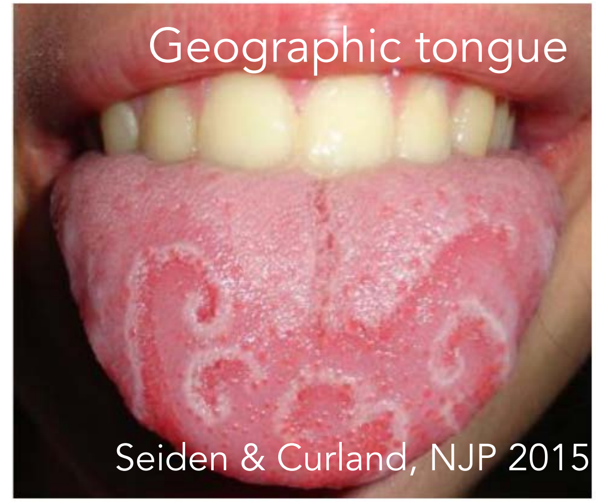
- Structure in non-equilibrium systems with fixed external condition

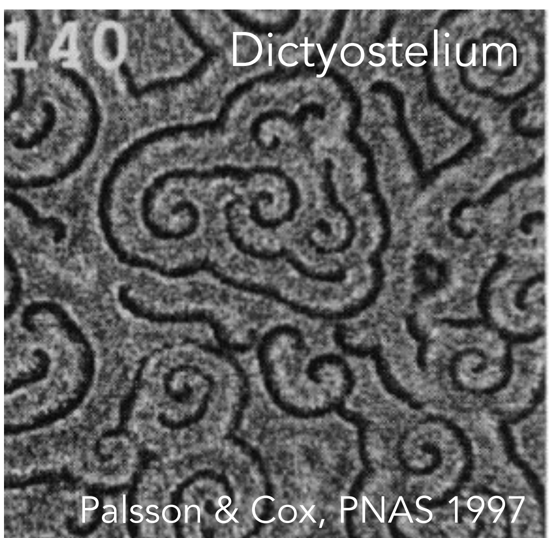
- No “free energy”

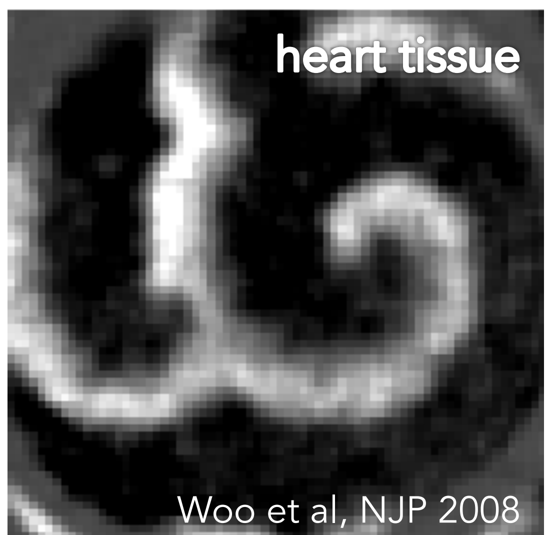
- Structure emerges from dynamics, nonlinear PDEs.

Pattern formation in excitable media

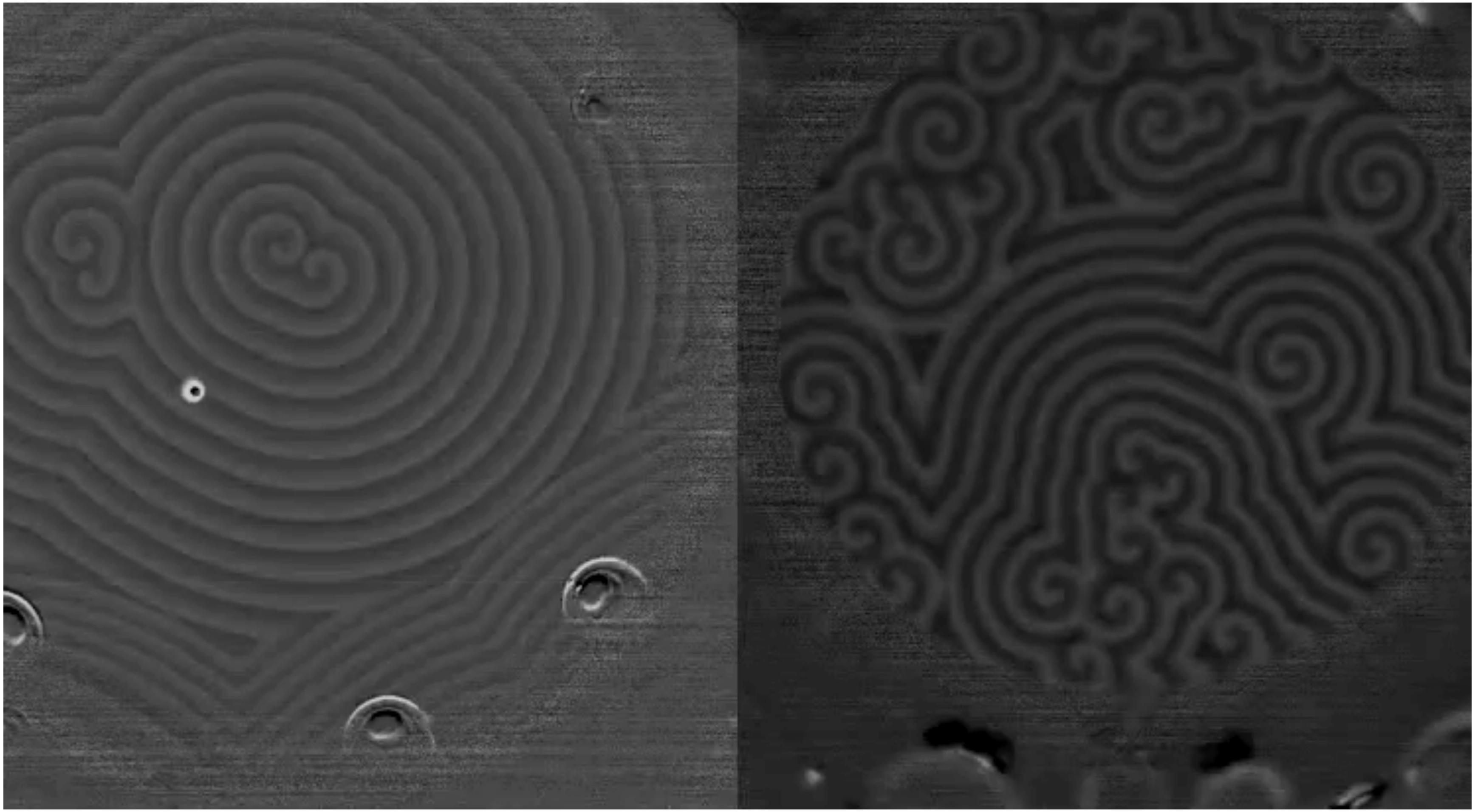






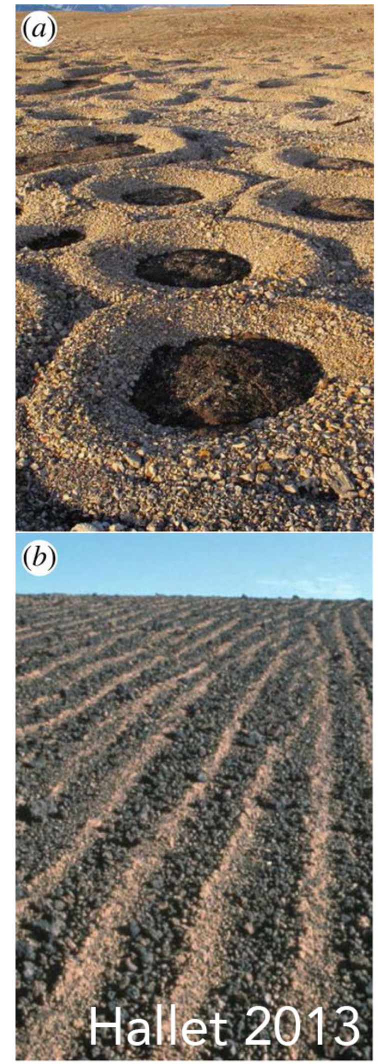


Pattern formation in excitable media



Pattern formation in geology

Pattern formation in geology



Goering & Morris, *PNAS* 2009

Hallet, *Phil. Trans. R. Soc.* 2013

Pattern formation in geology

Goering & Morris, *PNAS* 2009

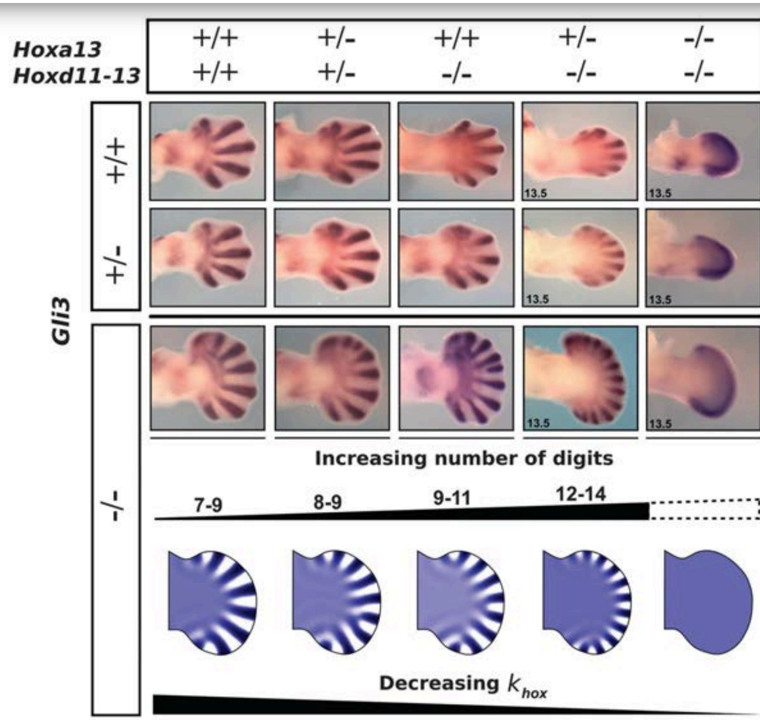
Stolum, *Science* 1996

Pattern formation in biology

The Chemical Basis of Morphogenesis

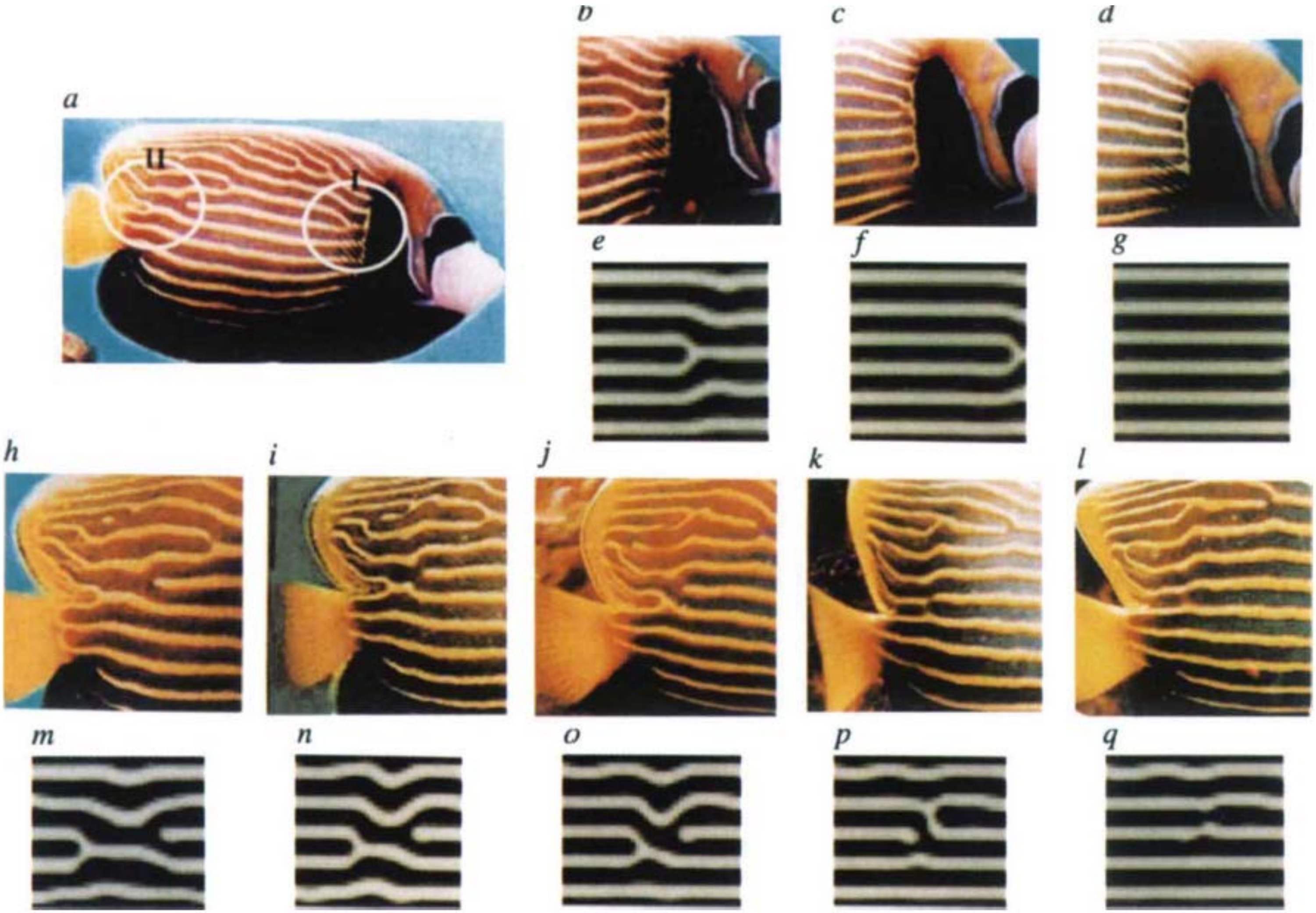
A. M. Turing

Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.



Rushikesh Sheth et al. *Science* 2012

Pattern formation in biology

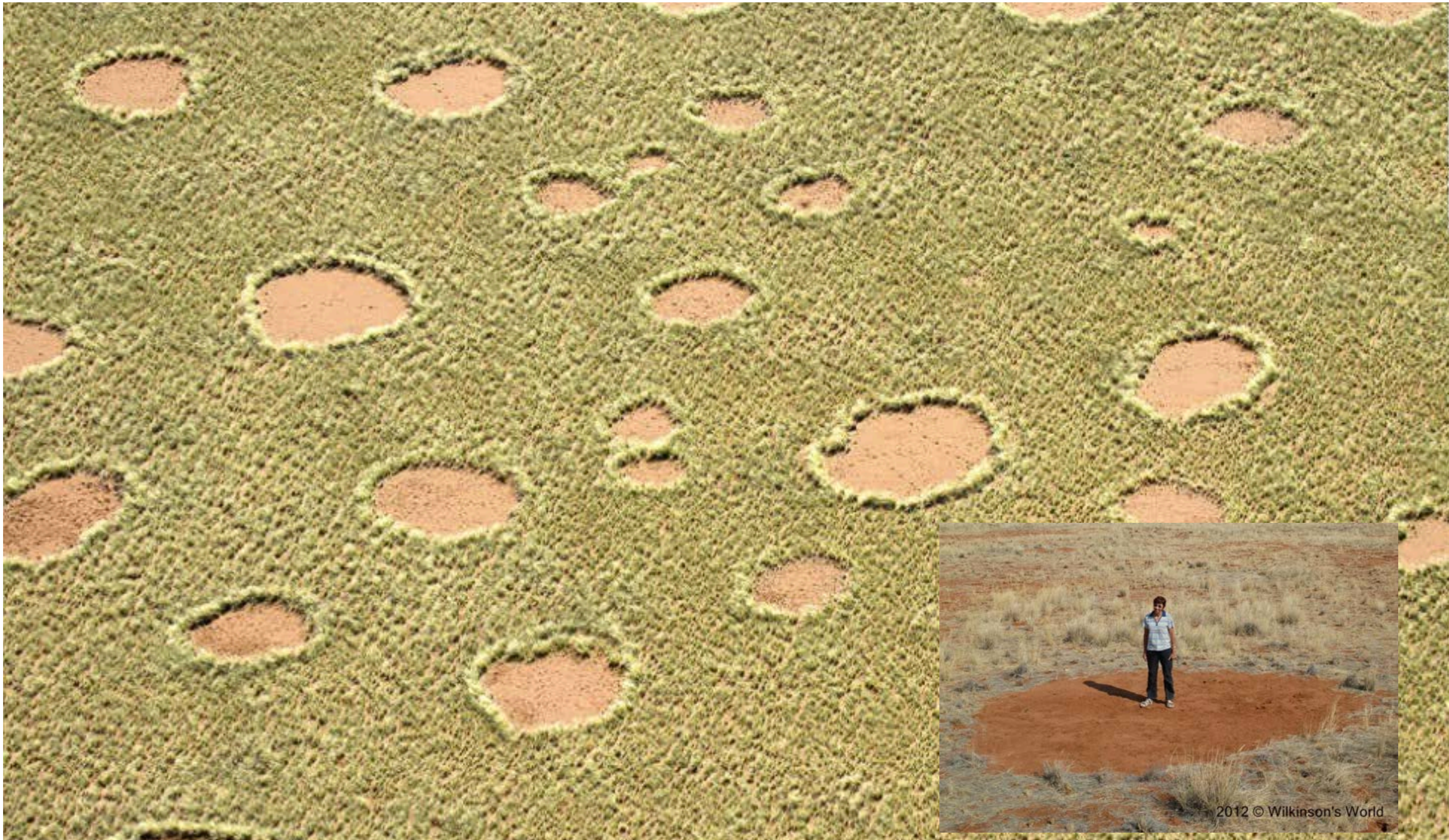


Kondo & Asai, *Nature* 1995

Pattern formation in biology

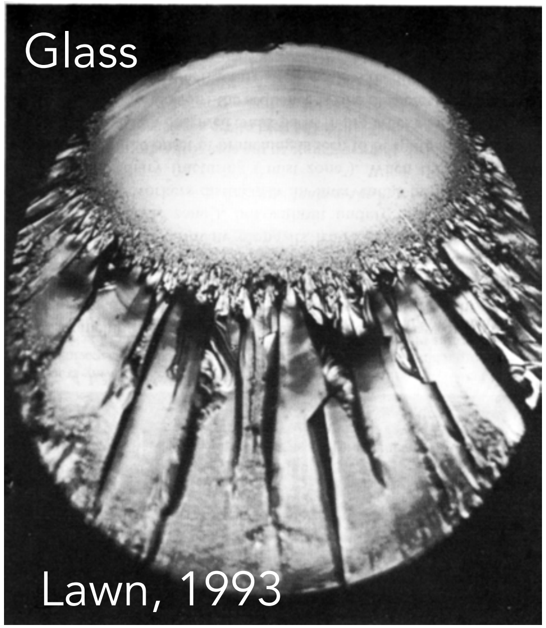
Meron et al, *Chaos, Solitons, and Fractals* 2004.

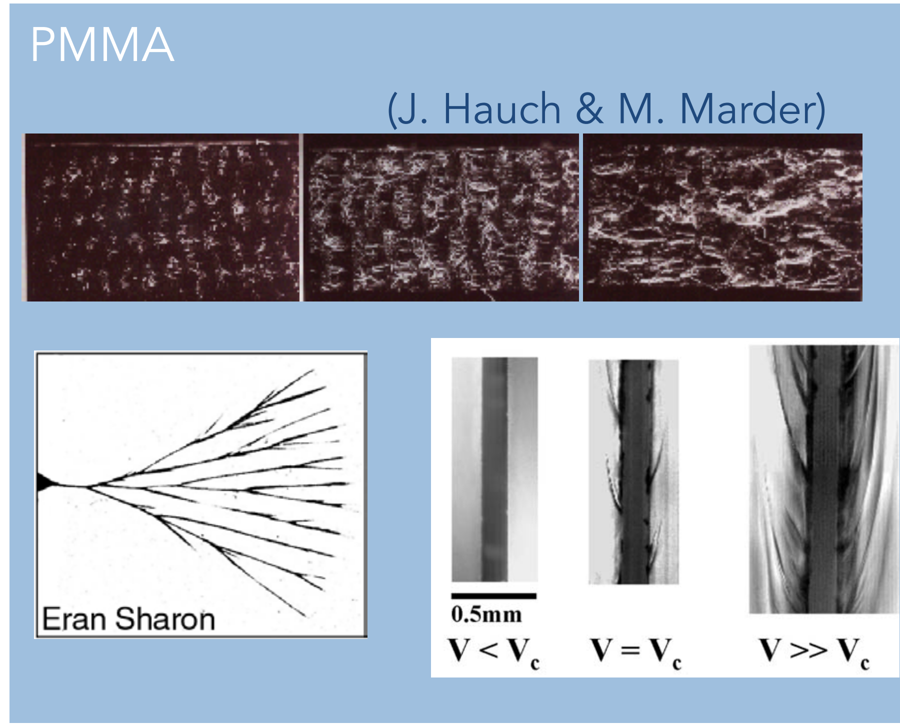
Pattern formation in biology

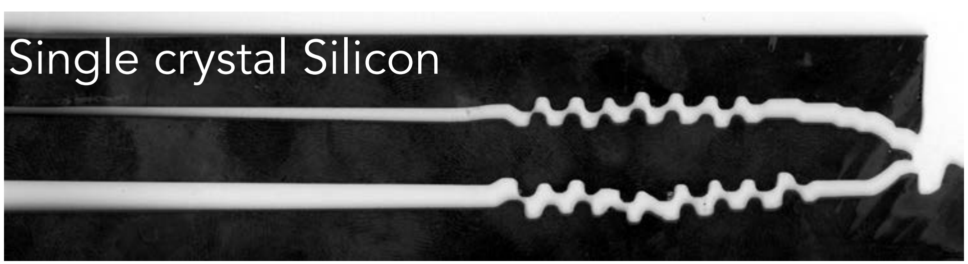


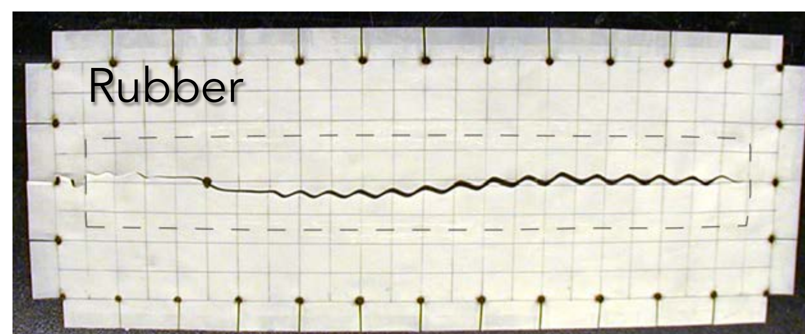
Zelnik et al, PNAS 2015

Pattern formation at fronts: fracture

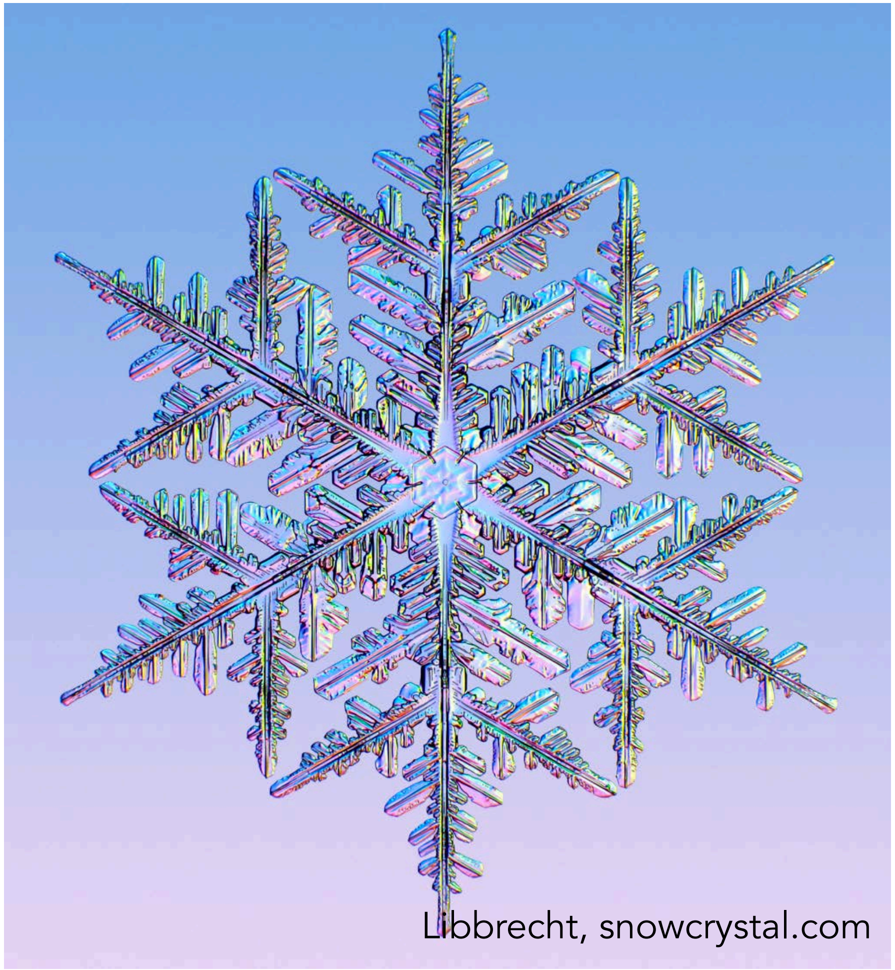








Pattern formation at fronts: fractals


Libbrecht, snowcrystal.com

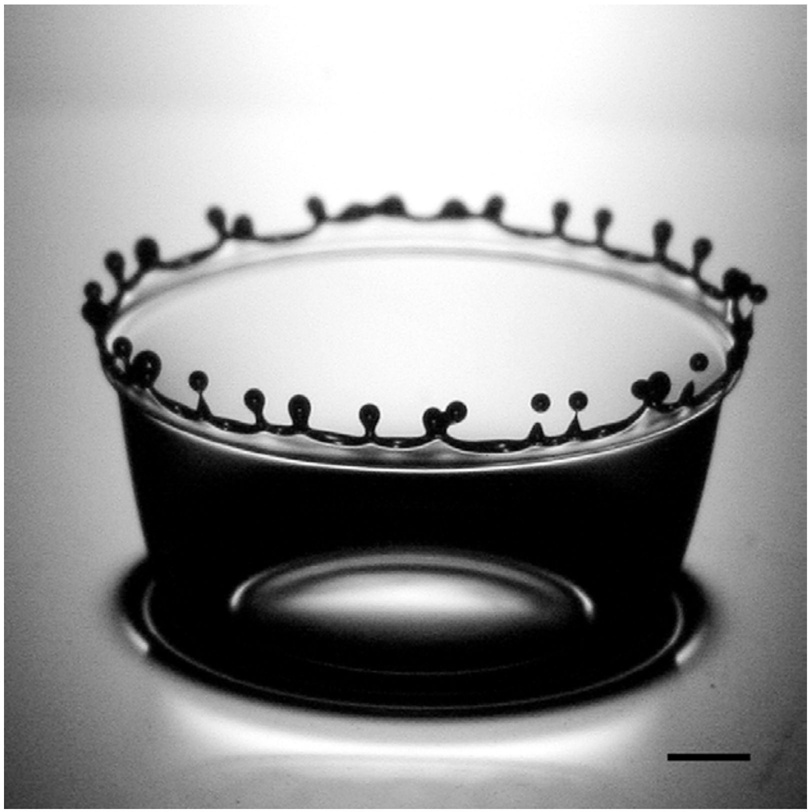
Viscous invasion

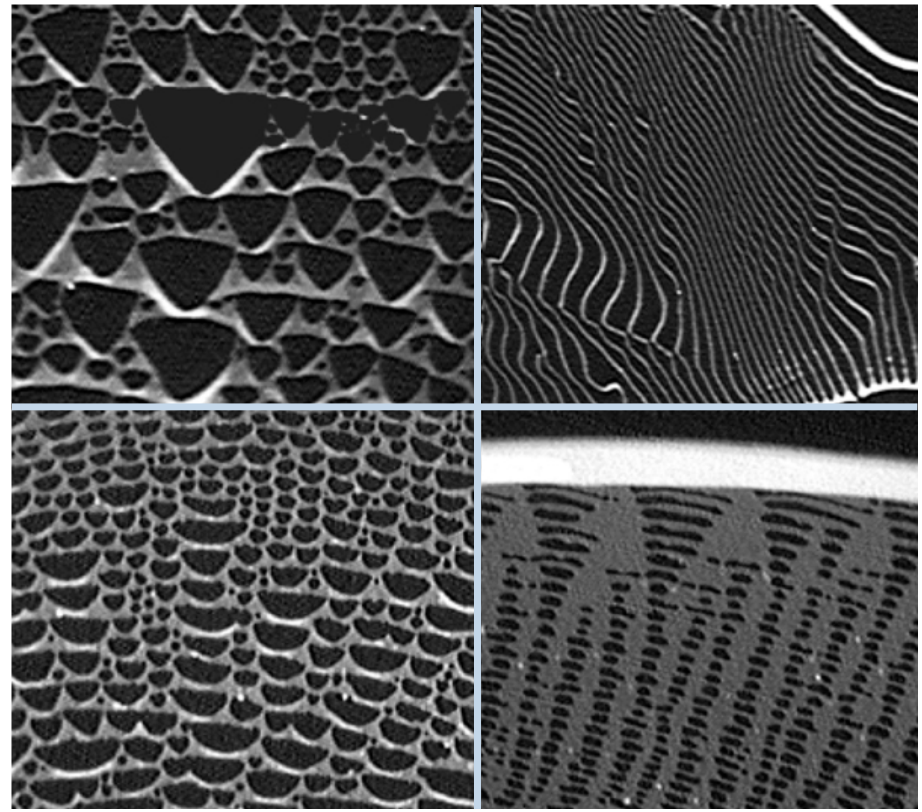

Praud & Swinney, *PRE* 2005


Dielectric breakdown

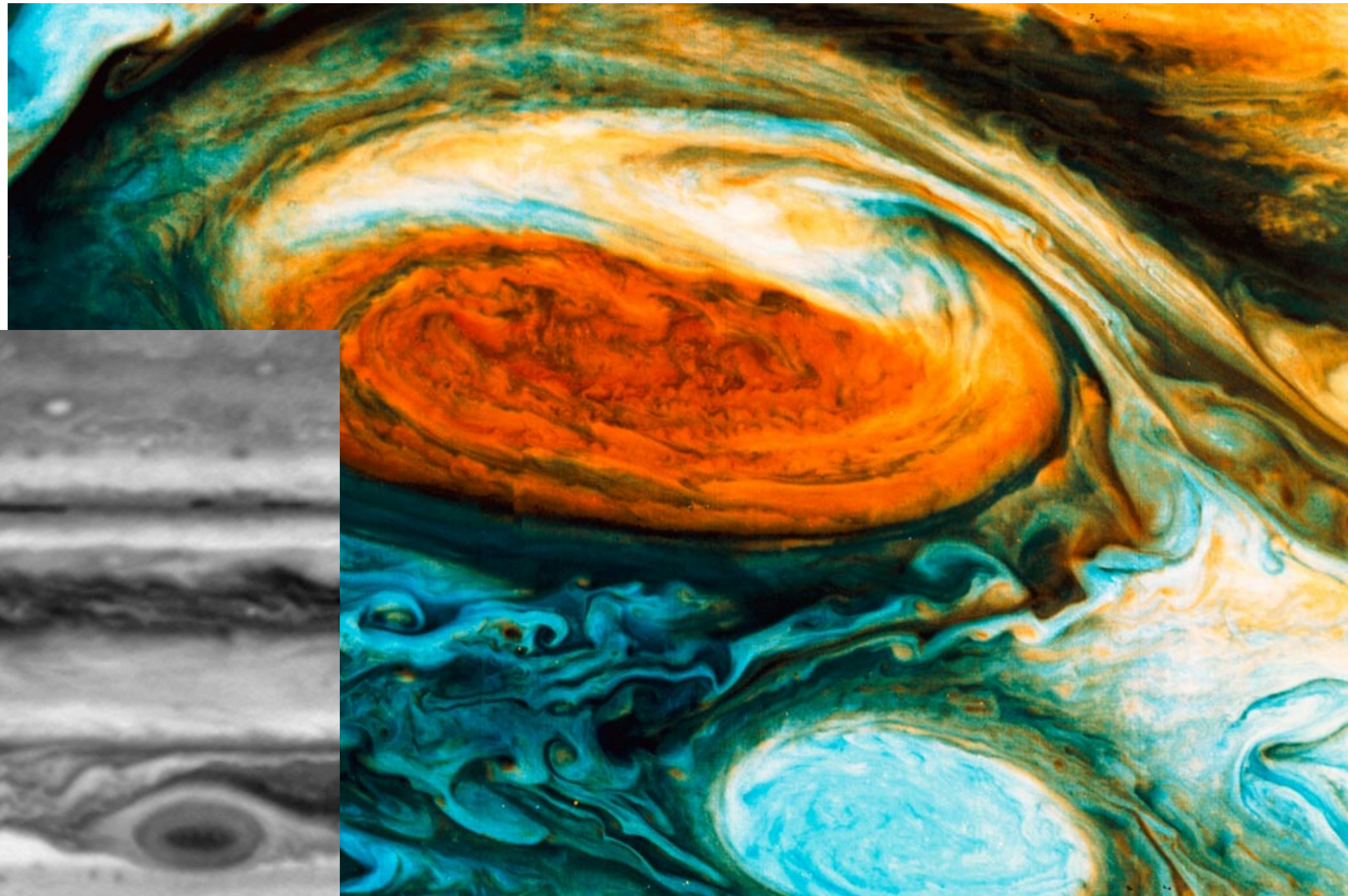
Linear stability is not so useful.
Doesn't saturate.

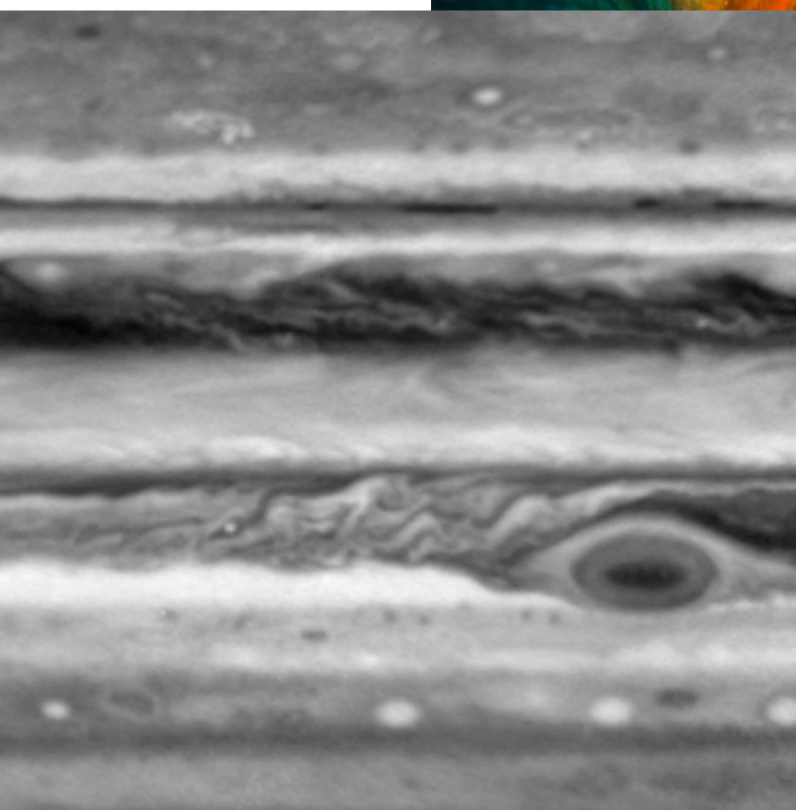
Pattern formation at fronts: capillary





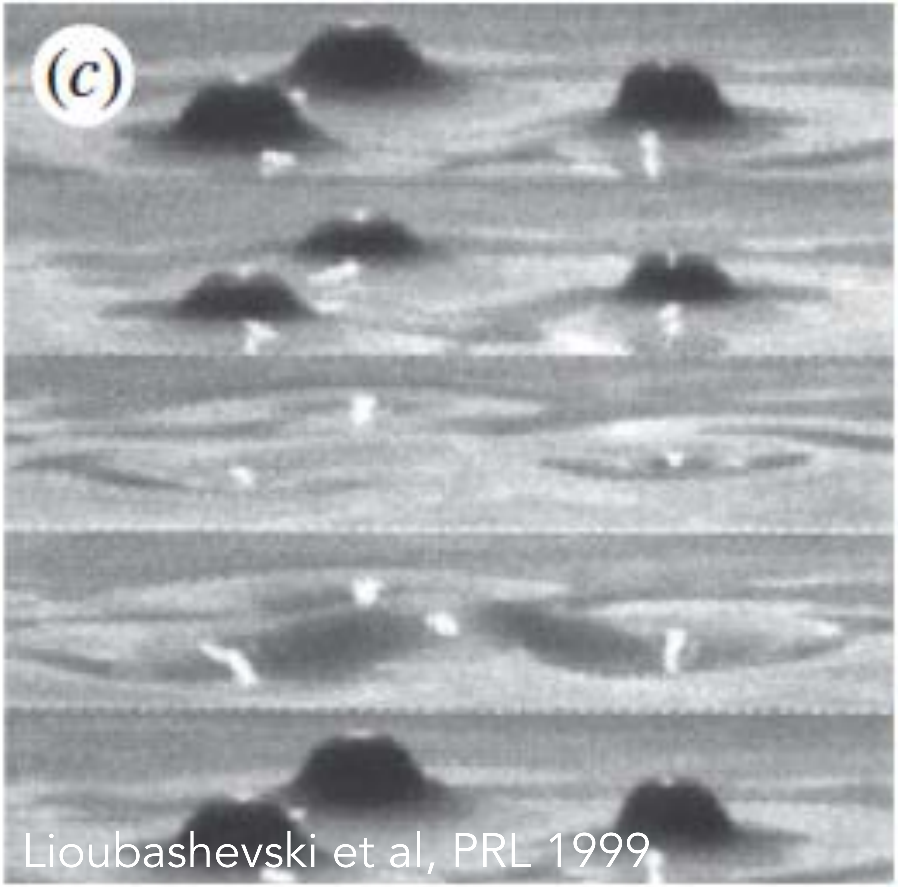
Localized states in pattern formation

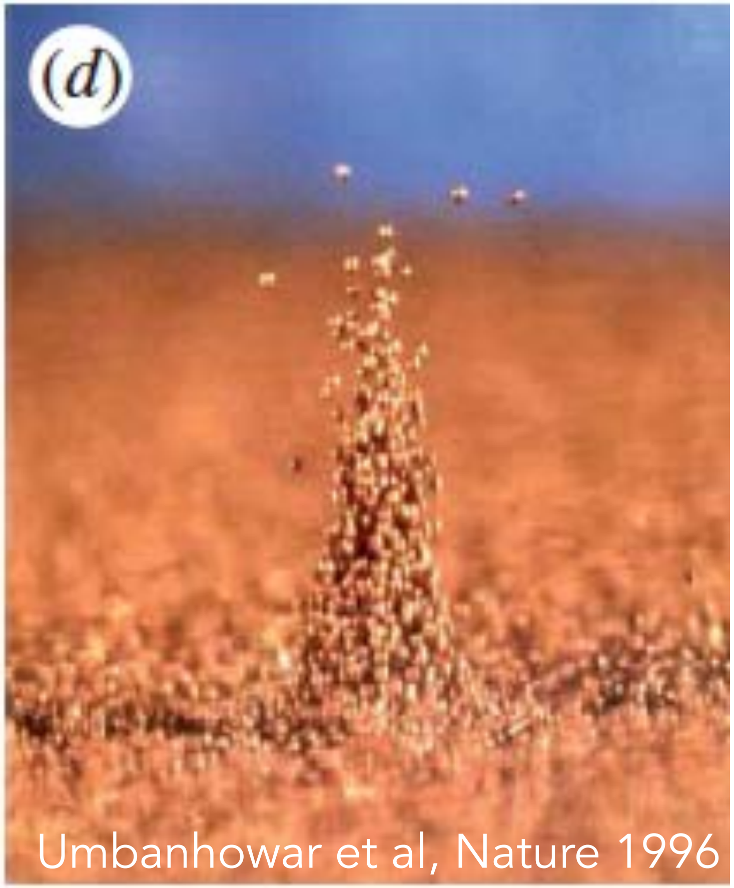




Localized states in pattern formation

Oscillons






Holes
Surface perturbed with jet of air
f=120 Hz
a=15 g

My background

- Experimentalist
- Particle deposits at contact lines
- Instabilities in fracture
- vibrated complex fluids
- BZ reaction
- splashing from drop impact

Outline

"The next great era of awakening of human intellect may well produce a method of understanding the qualitative content of equations…Today we cannot see that the water flow equations contains such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrodinger's equation contains frogs, musical composers, or morality--or whether it does not."

- R.P. Feynman in *The Feynman Lectures on Physics* (1964)

1. Qualitative theory of ODEs
2. Linear stability analysis
3. Weakly nonlinear analysis
4. Excitable systems
5. Parametrically driven systems

References

- Nonlinear dynamics
  - Strogatz, Nonlinear dynamics & chaos
- Linear stability
  - Drazin & Reed, Hydrodynamics stability
- Weakly nonlinear analysis
  - Bender & Orszag, Advanced mathematical methods for scientists and engineers: Asymptotic methods & perturbation theory
  - Godreche & Manneville, Hydrodynamics and nonlinear instabilities
- Pattern formation
  - Cross & Greenside, Pattern formation and dynamics in nonequilibrium systems
  - Cross & Hohenberg, Pattern formation outside of equilibrium