

Outline: L3

- * Mechanics & Newtonian mechanics
- * Newton's laws & equation of motion.
- * Central forces & central potentials
- * Types of potentials; & wtf Functions of one real variable.

Mechanics:

Suppose we have a system of particles, having coordinates \vec{r}_i ($i = 1, \dots, N$ (number of particles)). These particles

that do not feel interact with the rest of the world.

Mechanics provides a precise description of the dynamics of particles and systems of particles. In order to

mathematically describe the motion of these bodies over time, one needs a notion of the state:

state

time and position.

In Newtonian mechanics

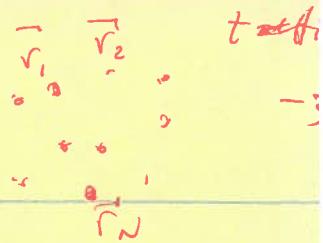
The state $S(t)$ of a system is typically a number, or a set of numbers, which tells us information about positions of the particles, $\vec{r}_i(t)$ and their velocities, $\vec{v}_i(t)$, at any given moment a given time, t . Here $i = 1, \dots, N$ where N is the # of particles.

So in 3D, $S(t) = \{\vec{r}_i(t), \vec{v}_i(t)\}, i=1, \dots, N$,

where velocities are related to coord 3D coordinates $\vec{r}_i(t)$ as

$$\vec{v}_i(t) = \frac{d\vec{r}_i(t)}{dt} = \vec{r}_i'(t)$$

Behinderer: Newton's Laws



I. Law: A body remains at rest or in uniform motion unless acted upon by a force.

IV Law: A body $\cancel{\text{act}} \Rightarrow \text{If } \sum_{j=1}^N \vec{F}_{ij} = 0,$

where \vec{F}_{ij} is the intera. force exerted by the j -th particle to the i -th particle,

then i -th particle remains at rest or in uniform motion.

II Law: $m_i \ddot{r}_i = \sum_{j=1}^N \vec{F}_{ij}$, - called equation of motion of a particle -

where m_i is a ~~chara~~ positive number characterizing the mass i -th particle called inertial mass.

From here we see that the acceleration

$$\vec{a}_i(t) \equiv \vec{r}_i(t) = \frac{d^2 \vec{r}_i(t)}{dt^2} = \dot{\vec{v}}(t)$$

is equal to

$$\vec{a}_i(t) = \frac{\sum_j \vec{F}_{ij}}{m_i}.$$

We are going to solve Newton's equations

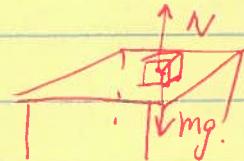
$$m_i \ddot{\vec{r}}_i = \sum_{j=1}^N \vec{F}_{ij} \quad i=1, \dots, N$$

for variety systems having different forms of \vec{F}_{ij} .

III Law: If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.

$$\vec{F}_1 = -\vec{F}_2$$

ex 1.



$$N = -mg$$

ex 2:



Defining momentum vector of a particle i

as

$\vec{P}_i \equiv m_i \vec{V}_i$, we can rewrite the Third

law as follows.

Using II law: $\vec{F}_1 = m_1 \vec{a}_1 = m_1 \frac{d\vec{V}_1}{dt} = \frac{d(m_1 \vec{V}_1)}{dt} = \frac{d\vec{P}_1}{dt}$

$$\vec{F}_2 = \frac{d\vec{P}_2}{dt}$$

So ~~etc~~ $\vec{F}_1 + \vec{F}_2 = 0 \Rightarrow \frac{d}{dt}(\vec{P}_1 + \vec{P}_2) = 0 \Rightarrow \vec{P}_1 + \vec{P}_2 = \text{constant vector.}$

Conservation of linear momentum

* Newton's equation of motion is in fact an approximation, that does not always work breaks down when

- m is a function of time.
- for velocities comparable to the speed of light (\rightarrow special relativity)
- for atomic and subatomic systems (\rightarrow quantum mechanics)
- when gravity is strong (\rightarrow general relativity)

\checkmark suppose we deal with classical systems
for which Newton's equation holds!

don't
deal
yet.

Moreover, if we make two more reasonable (rather realistic) assumptions:

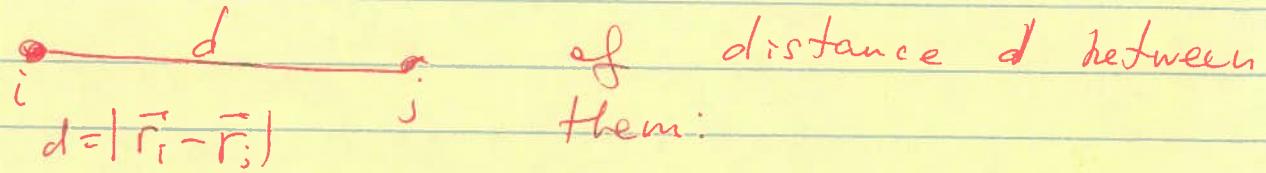
(i) require that $\vec{F}_{ij} = \vec{F}(\vec{r}_i, \vec{r}_j)$ - is a function of \vec{r}_i and \vec{r}_j only.

(ii) require that the forces are given by gradients of some scalar functions.

$$\vec{F}_{ij} = - \frac{\partial}{\partial \vec{r}_i} U_{ij}(\vec{r}_i, \vec{r}_j) :$$

Here $U_{ij}(r_i, r_j)$ - is called interaction potential and \vec{F}_{ij} - are called potential forces.

(iii) Finally, we require that the potential V_{ij} between two particles i, j is a function ^{interaction}



$$V_{ij}(\vec{r}_i, \vec{r}_j) = V_{ji}(\vec{r}_i, \vec{r}_j) \equiv V_{ij}(|\vec{r}_i - \vec{r}_j|) = V_{ij}(d)$$

Such a potential is called a central potential and corresponding forces are central forces.

Properties of central forces:

From $\vec{F}_{ij} = -\frac{\partial}{\partial \vec{r}_i} V_{ij}(|\vec{r}_i - \vec{r}_j|)$ it follows that

$$\begin{aligned}\vec{F}_{ij} &= -\frac{\partial}{\partial \vec{r}_i} V_{ij}(|\vec{r}_i - \vec{r}_j|) = -\frac{\partial |\vec{r}_i - \vec{r}_j|}{\partial \vec{r}_i} \cdot \frac{\partial V_{ij}(|\vec{r}_i - \vec{r}_j|)}{\partial |\vec{r}_i - \vec{r}_j|} \\ &= -\frac{\partial \sqrt{(\vec{r}_i - \vec{r}_j)^2}}{\partial \vec{r}_i} \cdot \frac{\partial V_{ij}(|\vec{r}_i - \vec{r}_j|)}{\partial |\vec{r}_i - \vec{r}_j|} \\ &= -\frac{\partial (\vec{r}_i - \vec{r}_j)}{\partial |\vec{r}_i - \vec{r}_j|} \cdot \frac{\partial V_{ij}(|\vec{r}_i - \vec{r}_j|)}{\partial |\vec{r}_i - \vec{r}_j|}\end{aligned}$$

so if we introduce a function

$$\varphi_{ij}(|\vec{r}|) = \varphi_{ij}(r) = -\frac{1}{r} \frac{d V_{ij}(r)}{d r}, \text{ then}$$

$$\vec{F}_{ij} = (\vec{r}_i - \vec{r}_j) \cdot \varphi_{ij}(|\vec{r}_i - \vec{r}_j|).$$

Implications:

\vec{F}_{ij} is parallel to $(\vec{r}_i - \vec{r}_j)$, and

$$\vec{F}_{ij} = -\vec{F}_{ji} \Leftrightarrow \text{Newton's 3-rd law!}$$

For central potentials Newton's equations of motion acquire the following form:

$$m \ddot{\vec{r}}_i = \sum_{j=1}^N \vec{F}_{ij} = -\frac{\partial}{\partial \vec{r}_i} \sum_{j=1}^N U_{ij} (\|\vec{r}_i - \vec{r}_j\|) \\ = (\vec{r}_i - \vec{r}_j) \cdot \nabla_{ij} U_{ij} (\|\vec{r}_i - \vec{r}_j\|)$$

represents a ^{ordinary} differential equation of second (ODE) of second order for a given central potential.

What kind of potentials do we have? Many!

* $V(r)$ is a function of one real variable.

* $V(r)$ maps one real number r to another real number $V(r)$. We write $V: \mathbb{R} \rightarrow \mathbb{R}$.

EXAMPLE: $f(x) = x^2$ maps

0 to 0	space of r	space of $V(r)$.
1 to 1		
2 to 4		
3 to 9		
4 to 16		

* A function is not necessarily defined for all values of its arguments:

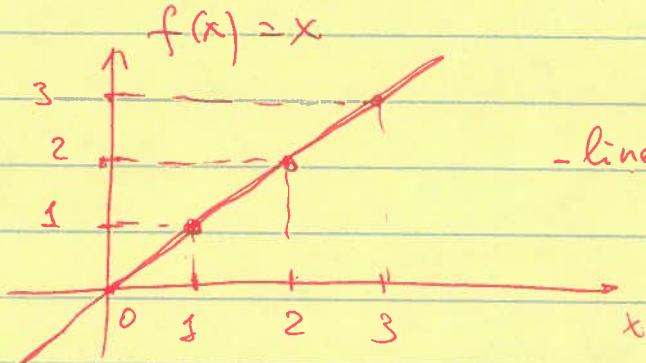
example: $f(x) = \sqrt{x}$ is not defined for $x < 0$.

The set of x for which $f(x)$ is defined is called domain of $f(x)$.

$$f: \begin{matrix} A \\ \downarrow \\ \text{domain} \end{matrix} \rightarrow B$$

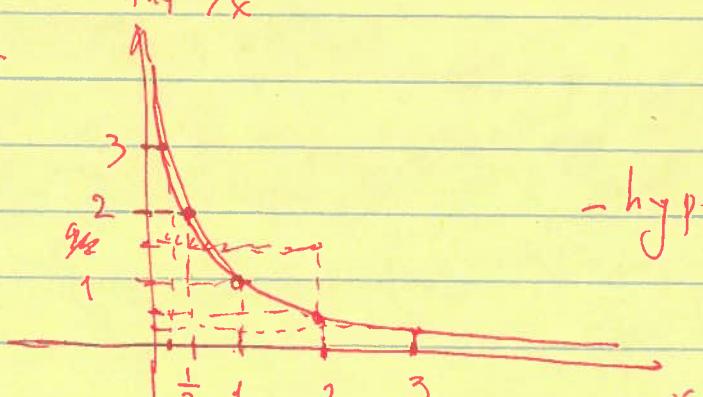
* The plot of a function gives a visual description of the variation of $f(x)$ as x changes.

example:

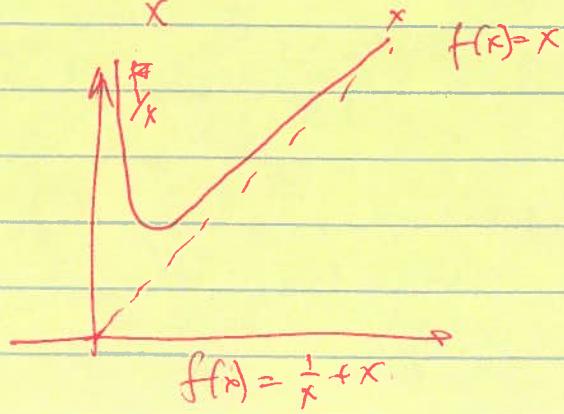
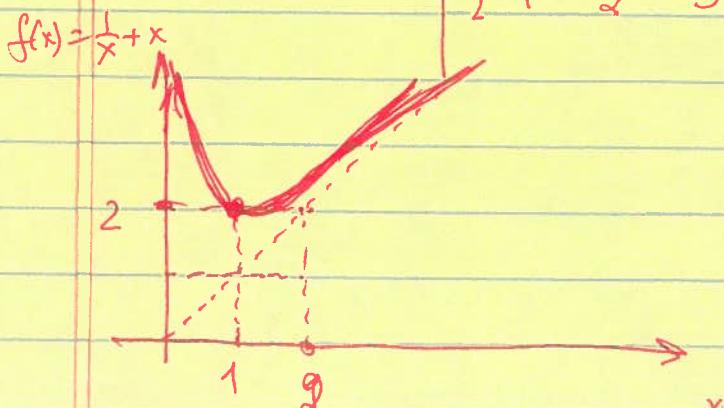


- linear function

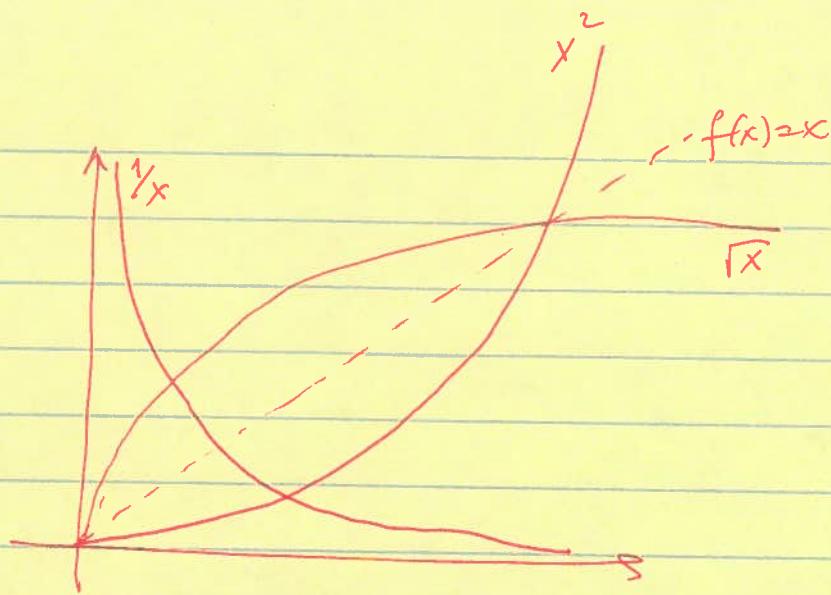
$$f(x) = \frac{1}{x}$$



- hyperbole

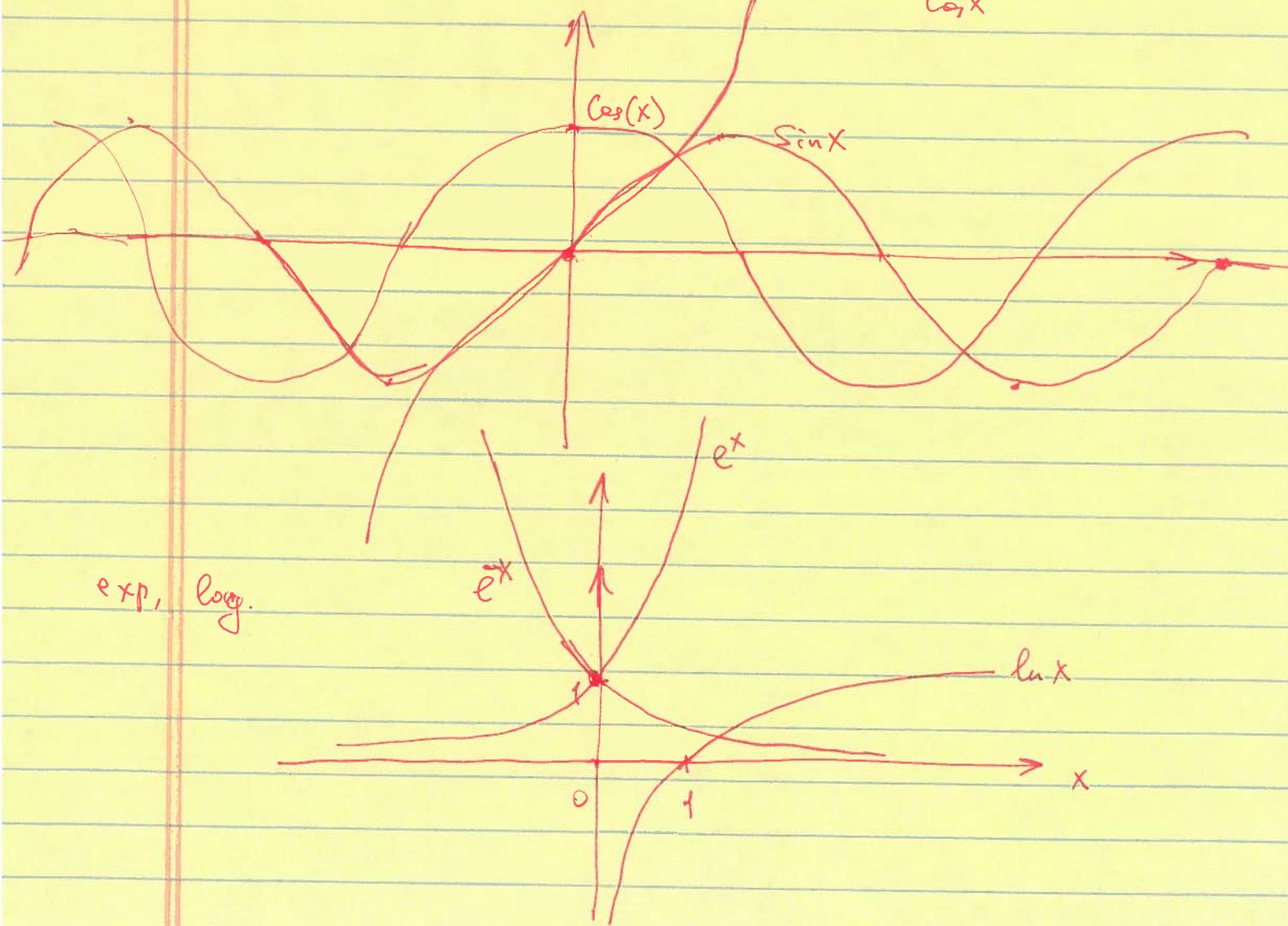


powers:

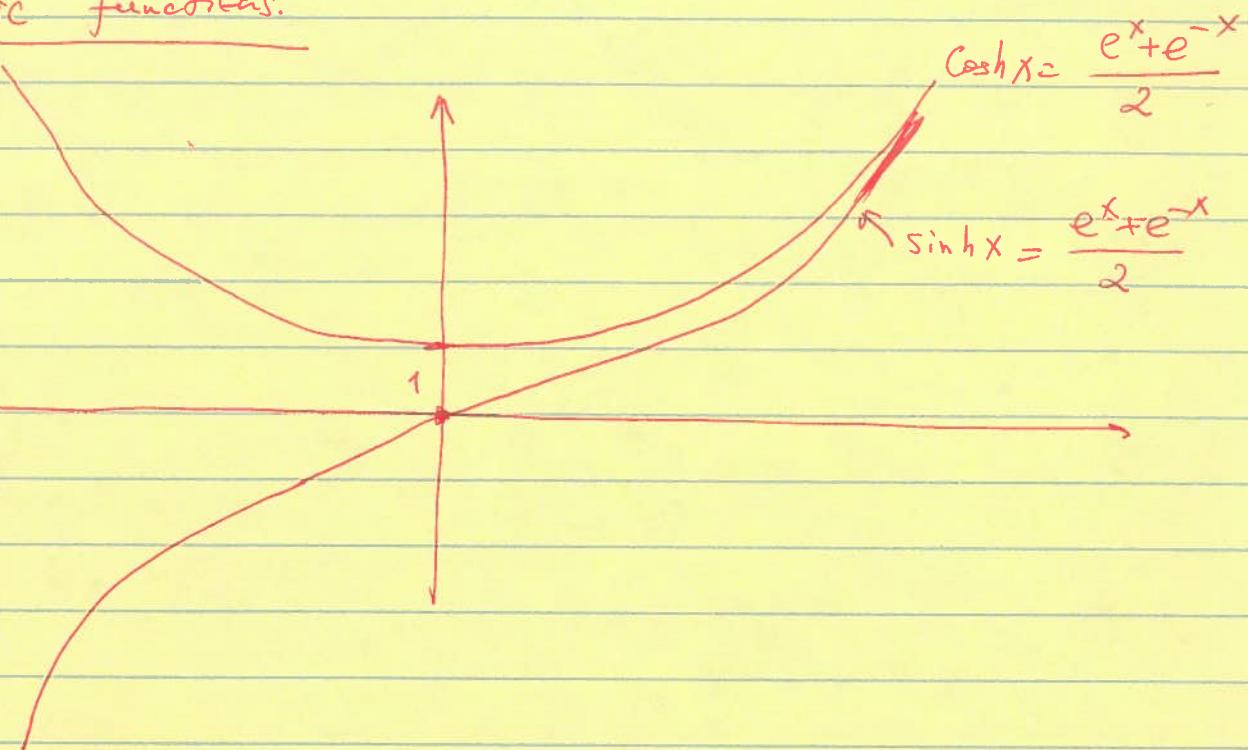


Trigonometric functions:

$$\tan x = \frac{\sin x}{\cos x}$$



Hyperbolic functions:



why do we use these notations \sinh , \cosh ?

$$\rightarrow e^{iy} = \cos y + i \sin y \quad \text{for } y \in \mathbb{R}$$

$$\text{then } \cos y = \frac{e^{iy} + e^{-iy}}{2}, \quad \sin y = \frac{e^{iy} - e^{-iy}}{2}$$

$$\begin{aligned} \text{So that } \cosh x &= \cos(ix) = \frac{e^{-x} + e^x}{2} \\ \sinh x &= -i \sin(ix) = -i \frac{e^{i(ix)} - e^{-i(ix)}}{2i} = \\ &= -\frac{e^{-x} - e^x}{2} = \frac{e^x - e^{-x}}{2}. \end{aligned}$$