PHY 421 FALL 2015 - FINAL EXAM

December 14th, 2015

Solve the following three problems. All the problems carry equal credit (but the questions inside each problem can have different weight). Books and notes are allowed.

1. A particle of mass m is subject to a central force $\vec{F}(\vec{r}) = F(r)\hat{r}$, where $F(r) = -\alpha/r^2 - \beta/r^3$, with α and β positive constants with appropriate dimensions.

(a) Compute the potential $V(r)$ associated with $\vec{F}(\vec{r})$. this force. Sketch a plot of $V(r)$. Then, compute the effective potential $V_{\text{eff}}(r)$ for a particle of mass m and angular momentum $\vec{\ell}$ subject to the force $\vec{F}$.

(b) For what values of $\ell \equiv |\vec{\ell}|$ is it possible to find stable bound orbits (i.e. orbits that do not reach infinity and do not fall on the origin)? For these values of ℓ, compute, as a function of α, β, m and ℓ the radius and the energy of the *circular* orbits.

(c) For the values of ℓ found in the previous question, let the particle have an arbitrary energy E. For which range of values of E does the particle move on a bound orbit? For these values of E, find the minimal r_{min} and the minimal r_{max} distance of the particle from the origin.

(d) For bound orbits, derive the expression $r(\theta)$ giving the shape of the orbit as a function of the parameters α, β, m and ℓ.

2. A particle of mass m is attached to the ceiling through a spring of negligible mass with spring constant k. A second particle, also with mass m, is attached to the first particle through a second spring, also of negligible mass, and also with constant k. The system is subject to gravity with gravitational acceleration $\vec{g}$, and both springs have zero length when left at rest.

(a) How many degrees of freedom does this system have? Choose an appropriate set of generalized coordinates and compute the lagrangian for this system. Then, derive the equations of motion for this system.

(b) Find the equilibrium position(s) for the system. Are they stable?

3. A particle of mass m moves along the x-direction and is subject to a velocity-dependent frictional force with expression $F(v) = -b\sqrt{|v|}$ where $v \equiv \dot{x}$.

(a) Find $v(t)$ if $v(t=0) = v_0 > 0$.

(b) At what time does the particle stop?

(c) Setting $x(t=0) = 0$, find $x(t)$. How far will the particle travel before it stops?

(d) Re-derive the result of (c), up to numerical coefficients, without solving any differential equation, but by simply using dimensional analysis.