PHY 421 FALL 2016 - FINAL EXAM

Solve the following problems. All the problems carry equal credit (but the questions inside each problem can have different weight). Books and notes are allowed. No electronic devices, except for calculators, can be used.

1. Use the general solution $x(t)$ to the differential equation $\ddot{x}(t) + 2\beta\dot{x}(t) + \omega_0^2 x = 0$, with constant β and ω_0, of underdamped motion and choose the constants of integration to satisfy the initial conditions $x(t = 0) = x_0$ and $v(t = 0) = v_0$.

2. Find the angular frequency, ω, of one-dimensional small oscillations in the potential field $U(x) = U_0 \sin(\alpha x) - F \cdot x$. Here U_0 and F are constants.

3. The following Hamiltonian describes light propagation through a medium in 3D

$$H(\mathbf{p}, \mathbf{r}) = \frac{c|\mathbf{p}|}{n(\mathbf{r})}, \tag{1}$$

where c is the speed of light and $n(\mathbf{r})$ is a function of $\mathbf{r} = \overrightarrow{(x, y, z)}$ that is called refractive index. (a) Find Hamilton equations for a general function $n(\mathbf{r})$. (b) Consider the case when $n(\mathbf{r}) = ax$, where a is a constant and x is the projection of $\mathbf{r}$ on the x-axis. Simplify all Hamilton equations in case of this refractive index.