

PHY 421 FALL 2016 - MIDTERM EXAM 1

Solve the following problems. All the problems carry equal credit (but the questions inside each problem can have different weight). Books and notes are allowed. No electronic devices, except for calculators, can be used.

1. Show that

$$\nabla f(r) = \frac{\mathbf{r}}{r} \frac{df(r)}{dr}, \quad (1)$$

where \mathbf{r} is a three dimensional vector, $r = |\mathbf{r}|$ is its magnitude, and $f = f(r)$ is a function of r .

2. Suppose a point particle is moving in one-dimension in the region $x > 0$. The particle is influenced by the potential $U(x)$, where

$$U(x) = U_0 \left(\frac{a}{x} + \frac{x}{a} \right). \quad (2)$$

Here $U_0 = 1J$ and $a = 2m$. (a) Plot the potential. (b) Find the equilibrium points and determine whether they correspond to minima or maxima of the potential. (c) Find the force, $F(x)$, acting on the particle.

3. Consider a damped harmonic oscillator in underdamped regime. After four cycles the amplitude of the oscillator has dropped to $1/e$ of its initial value. Find the ratio of the frequency of the damped oscillator to its natural (without damping) frequency.