

PHY 421 FALL 2016 - MIDTERM EXAM 2

Solve the following problems. All the problems carry equal credit (but the questions inside each problem can have different weight). Books and notes are allowed. No electronic devices, except for calculators, can be used.

1. **Calculus of Variations:** Show that the shortest distance between two points (x_1, y_1) and (x_2, y_2) , on a plane is a straight line (recall: the segment of the curve given by $y = f(x)$ between these two points has the length $S[f] = \int_{x_1}^{x_2} g dx$, where $g = \sqrt{1 + (f'(x))^2}$).

2. A particle moves in a plane under the influence of the force $F = -Ar^{\alpha-1}$ directed toward the origin, where A and α are positive constants. (a) Choose generalized polar coordinates (r, θ) and let the potential energy be zero at the origin. Find the Lagrangian, Taylor-expand it in the vicinity of small $\theta \ll 1$, and write Euler-Lagrange equations of motion. (b) Show that the angular momentum, l_0 , about the origin is conserved (recall the angular momentum is $l = mr^2\dot{\theta}$). (c) Using conservation of the angular momentum obtain differential equations for the radius vector, $r(t)$, that does not include the other generalized coordinate, $\theta(t)$.

3. A particle moves in a central potential field $U(r) = -Ve^{-kr}$, with positive constant k and V . For what values of the angular momentum l_0 a bounded motion of the particle (i.e. when the particle does not escape to the infinity) is possible?