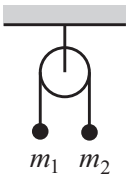
PHY 421 FALL 2017 - FINAL EXAM

Solve the following problems. All the problems carry equal credit (but the questions inside each problem can have different weight). Books and notes are allowed. No electronic devices, except for calculators, can be used.

1. Find the angular frequency, ω, of one-dimensional small oscillations of the point-like particle of mass m in the potential field $U(x) = U_0 e^{\alpha x} - Fx$. Here U_0, α, and F are positive constants.

2. The mechanical setup you are asked to study is shown in figure. Define the vertical coordinate of the left object with mass m_1 by y (here one can assume that e.g. the upward direction is positive). a) Find the Lagrangian $\mathcal{L}(y, \dot{y})$ and then the Hamiltonian $H(y, p_y)$ describing the system, where p_y is the momentum that corresponds to y. b) Write down the Hamilton's equations of motion.

Hint: estimate the potential energy when m_1 has a vertical coordinate y. Notice that if m_1 moves up by y, then m_2 moves down by y.



3. Two pointlike objects of mass m are connected by a spring. The spring constant is k and its relaxed length is l. The particles are free to move along a frictionless horizontal wire. Let their positions are x_1 and x_2. a) Find the Lagrangian $\mathcal{L}(x_1, x_2; \dot{x}_1, \dot{x}_2)$. b) Find the Hamiltonian $H(x_1, x_2; p_1, p_2)$. c) Write down the four Hamilton's equations.

Hint: $H = p_1\dot{x}_1 + p_2\dot{x}_2 - \mathcal{L}$.