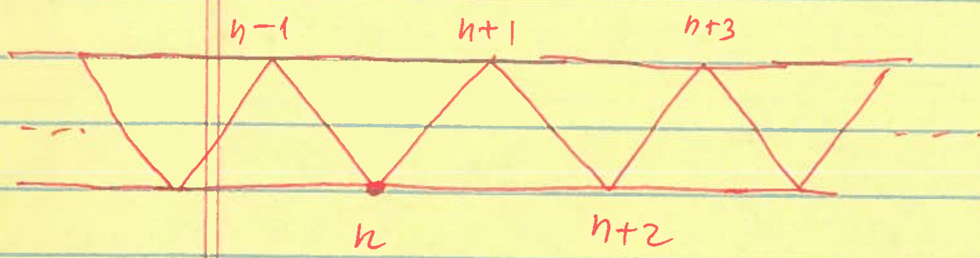
Homework 1:

P. 1. (5 points): Consider a zig-zag ladder chain



with Pauli matrices residing on lattice sites:
$\vec{\sigma}_n = (\sigma_n^x, \sigma_n^y, \sigma_n^z)$, that define $s=1/2$ operators.

Let us construct a chirality operator:

$$\chi_{n,n+1,n+2} = \vec{\sigma}_n \cdot \left(\vec{\sigma}_{n+1} \times \vec{\sigma}_{n+2} \right).$$

a) Show that $\hat{H}_\chi = \sum_{n \in \text{lattice}} \chi_{n,n+1,n+2}$ commutes with the Heisenberg Hamiltonian

$$\hat{H}_{XXX} = \sum_n \vec{\sigma}_n \cdot \vec{\sigma}_{n+1}.$$

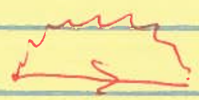
b) Express $\hat{H}_{\chi^2} = \sum_n \chi_{n-1,n,n+1} \cdot \chi_{n,n+1,n+2}$ in terms of scalar products of type $(\vec{\sigma}_a \cdot \vec{\sigma}_b)$ and pseudoscalar χ_{abc}-s.

c) Let us introduce Jordan-Wigner fermions as

$$c_k^+ = \sigma_k^+ \prod_{n=-\infty}^{k-1} \sigma_n^z, \qquad c_k = \sigma_k^- \prod_{n=-\infty}^{k-1} \sigma_n^z .$$

Find the fermion representation of $\chi_{k,k+1,k+2}$.

P.2. (5 points): In Lecture 2 of Week 3 we have calculated the self-energy $\Sigma(\varepsilon, \vec{p})$ of a polaron in first order in electron-phonon interaction.

$\Sigma(\varepsilon, \vec{p}) =$ . The obtained self-energy $\Sigma(\varepsilon, p) =$

$= \mathrm{Re}\,\Sigma + i \cdot \mathrm{Im}\,\Sigma$, has a finite $\mathrm{Im}\,\Sigma$-part when $v = \frac{p}{m} > c$, because such a fast electron can emit phonons (Cherenkov effect).

Express $\mathrm{Im}\,\Sigma$ as an angular integral

$\mathrm{Im}\,\Sigma = \int \Gamma(\theta)\, d\theta$, where θ is the angle between electron's momentum, $\vec{p}$, and phonon momentum $\vec{k}$. Here $\Gamma(\theta)$ will represent the angular distribution of intensity of the emitted "sound".