Week 4

* The thermodynamic limit: $L \to \infty$
* Phonons in a crystal
* The continuum limit: a (lattice constant) $\to 0$
* Example: 1D string
* Phonon propagator
* Bound states of two-particles
  - Interaction potential with no retardation $\left[U(\vec{r},t) = \bar{U}(\vec{r}) \cdot \delta(t) \right]$
  - Bethe-Salpeter equation
  - Reduction to a single-particle picture.

The thermodynamic limit: $L \to \infty$

By restricting a system to a finite lattice one imposes a restriction on the maximum wave-length => the excitation spectrum. This is called an "infrared cut-off." When we take $L \to \infty$, the allowed momentum states become closer to each other and one obtains a continuum of states in momentum space.

In each dimension the increment in momentum is

$$\Delta q = \frac{2\pi}{L} \Rightarrow L \cdot \frac{\Delta q}{2\pi} = 1$$ and the summation

can be written as:

$$\sum_{q_j} \{\dots\} = L \cdot \sum_{q_j} \frac{\Delta q}{2\pi} \{\dots\} \quad , \quad q_j = \frac{2\pi j}{L} \text{ and } j = 1, \dots, N.$$

When one takes the limit $L \to \infty$, q becomes continuous $q \in [0, 2\pi/a]$, $a = \frac{L}{N}$ - is the lattice spacing.

=> the summation can now be replaced by an integral:

$$\sum_{q} \{\dots\} \to L \int_0^{2\pi/L} \frac{dq}{2\pi} \{\dots\}$$

- for each dimension.

In D-dimensions: $(\Delta q)^D = \left(\frac{2\pi}{L}\right)^D$ and

$$\sum_q \{\cdots\} = L^D \sum_q \frac{(\Delta q)^D}{(2\pi)^D} \{\cdots\} \to L^D \int_0^{2\pi/a} dq_1 \cdots \int_0^{2\pi/a} dq_D \{\cdots\}$$

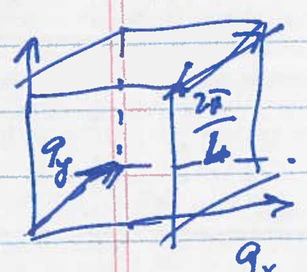
Example: Consider a 3D harmonic crystal

$$H = \sum_j \frac{\pi_j^2}{2m} + \sum_j \sum_{\vec{a}=(\vec{e}_x,\vec{e}_y,\vec{e}_z)} \frac{m\omega_0^2}{2}\left(\phi_j - \phi_{j+\vec{a}}\right)^2$$

where $\phi_j = \phi(x_j)$, $\pi_j = \pi(x_j)$ denote canonically conjugate (scalar) displacement and momenta at site j and $\vec{a}$ ~~$= (\vec e_x, \vec e_y, \vec e_z)$~~ is a unit vector separating nearest neighbor atoms

writing
$$\phi_j = \frac{1}{\sqrt{N}} \sum_{\vec{q}} \phi_{\vec{q}}\, e^{i\vec{q}\cdot\vec{x}}$$
$$\pi_j = \frac{1}{\sqrt{N}} \sum_{\vec{q}} \pi_{\vec{q}}\, e^{i\vec{q}\cdot\vec{x}},$$

$\vec{q} = \frac{2\pi}{L}(i,j,k)$ are the discrete momenta of a cubic crystal of volume L^3, and defining



$$b_{\vec{q}} = \sqrt{\frac{m\omega_q}{2\hbar}}\left(\phi_{\vec{q}} + \frac{i}{m\omega_q}\pi_{\vec{q}}\right),$$

$$b^{+}_{\vec{q}} = \sqrt{\frac{m\omega_q}{2\hbar}}\left(\phi_{-\vec{q}} - \frac{i}{m\omega_{\vec{q}}}\pi_{-\vec{q}}\right),$$

one reduces the Hamiltonian to its standard form:

$$H = \sum_{\vec{q}} \hbar \omega_{\vec{q}} \left(\hat{n}_q + \tfrac{1}{2} \right) \quad , \quad \hat{n}_q = \hat{b}_q^{+} \hat{b}_q \ -$$

is the phonon # operator,

and in 3D:

$$\omega_q = 2\omega_0 \left[\sin^2\left(\frac{q_x a}{2}\right) + \sin^2\left(\frac{q_y a}{2}\right) + \sin^2\left(\frac{q_z a}{2}\right) \right]^{1/2}.$$

The ground state energy corresponds to $n_{\vec{q}} = 0$

=> i.e. zero-point energy:

$$E_0 = \sum_{\vec{q}} \frac{\hbar \omega_q}{2} \xrightarrow[L \to \infty]{} V \cdot \int \frac{d^3 q}{(2\pi)^3} \frac{\hbar \omega_q}{2} ,$$

where $V = L^3$. Substituting for $\omega_{\vec{q}}$ we find

$$E_0 = V \left(\prod_{e=1,2,3} \int_0^{2\pi/a} \frac{d q_e}{2\pi} \right) \hbar \omega_0 \sqrt{\sum_{e=1,2,3} \sin^2\left(\frac{q_e a}{2}\right)} =$$

$$= I_3 \cdot N \hbar \omega_0 \ , \quad \text{where}$$

$$I_3 = \frac{1}{\pi^3} \int_0^{\pi} du_1 \int_0^{\pi} du_2 \int_0^{\pi} du_3 \sqrt{\sin^2 u_1 + \sin^2 u_2 + \sin^2 u_3} = 1{,}19.$$

$$\Rightarrow \quad E_0 = 1{,}19\, N \cdot \hbar \omega_0 .$$

The continuum limit: $a \to 0$

In contrast to $L \to \infty$, here we remove the discrete character of the system, allowing fluctuations of arbitrarily small wavelength and hence arbitrarily large energy.

For discrete system (periodic boundary cond):

$$k_{x,y,z} \lesssim \frac{2\pi}{a} \to \infty \Rightarrow \text{ultraviolet cut-off goes to infinity.}$$

In the continuum limit the plane-wave basis is

$$\langle \vec{x} | \vec{q} \rangle \longleftrightarrow e^{i \vec{q}\,\vec{x}}$$

and orthogonality condition is

$$\langle \vec{q}\,' | \vec{q} \rangle = \int d^D x \, e^{i(\vec{q}-\vec{q}\,')\cdot\vec{x}} = L^D \delta_{\vec{q},\vec{q}\,'}$$

If we also take thermodynamic limit:

$$L^D \delta_{\vec{q},\vec{q}\,'} = (2\pi)^D \frac{\delta_{qq'}}{(\Delta q)^D} \longrightarrow (2\pi)^D \delta^D(\vec{q}-\vec{q}\,')$$

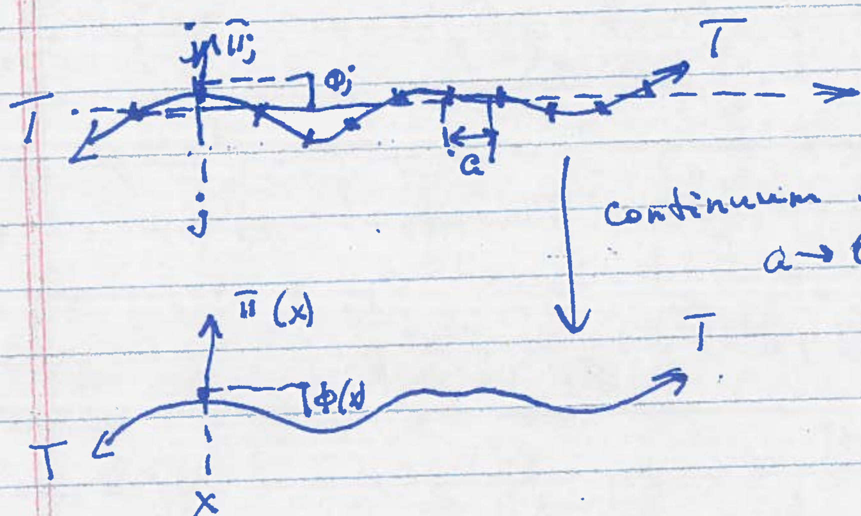
so the orthogonality condition becomes

$$\int d^D x \, e^{i(\vec{q}-\vec{q}\,')\cdot\vec{x}} = (2\pi)^D \delta^D(\vec{q}-\vec{q}\,').$$

Example: 1D string

Let us see that the continuum model and the discrete model have the same long-wavelength physics. Their behavior will only differ at very short distances, at high frequencies, over short times.

This is very simple example of renormalization.



The Hamiltonian of the discrete system is

$$H = \sum_{j=1}^{N} \left[\frac{\pi_j^2}{2m} + \frac{T}{2a} \left(\phi_j - \phi_{j+1} \right)^2 \right], \text{ where } \frac{T}{a} \to m\omega^2.$$

Continuum limit $\Rightarrow$ $a \to 0$ preserving $\rho = \frac{m}{a}$ and

$$\Rightarrow a \sum_j \to \int dx \, , \quad \frac{(\phi_j - \phi_{j+1})^2}{a^2} \to \left(\nabla_j \phi(x) \right)^2.$$

Making replacement $\frac{\pi_j}{a} \to \pi(x_j)$, we obtain

$$H = \int dx \left[\frac{I}{2} (\nabla_x \phi)^2 + \frac{1}{2\rho} \pi(x)^2 \right].$$

On the discrete lattice we have

$$[\phi(x_i), \pi(x_j)] = i\hbar \tilde{\delta}(x_i - x_j), \text{ where}$$

$\tilde{\delta}(x_i - x_j) = a^{-1} \delta_{ij}$. In the limit $a \to 0$, $\tilde{\delta}(x_i - x_j)$ behaves as a Dirac δ-function $\Rightarrow$

$$\Rightarrow [\phi(x), \pi(y)] = i\hbar\, \delta(x-y)$$ - continuum limit.

Notice here we can retain finite L.
In the thermodynamic limit $L \to \infty$, we obtain

$$\phi_q = \int dx\, \phi(x)\, e^{-iqx}$$

$$\pi_q = \int dx\, \pi(x)\, e^{-iqx}$$

$$\Rightarrow [\phi_q, \pi_{q'}] = \int dx dx'\, e^{-i(qx - q'x')} \overbrace{[\phi(x)\, \pi(x')]}^{i\hbar\,\delta(x-x')}$$

$$= i\hbar \underbrace{\int dx\, e^{-i(q-q')x}}_{\langle q|q'\rangle = 2\pi\delta(q-q')} = i\hbar \cdot 2\pi\delta(q-q')$$

- canonical commutation relations

Now repeating previous steps:

$$\phi(x) = \int \frac{dq}{2\pi} \phi_q \, e^{iqx}$$

$$\pi(x) = \int \frac{dq}{2\pi} \pi_q \, e^{iqx}$$, we obtain for the

Hamiltonian

$$H = \int \frac{dq}{2\pi} \left[\frac{\pi_q \pi_{-q}}{2\rho} + \frac{Tq^2}{2} \phi_q \phi_{-q} \right],$$

where the main effect is the replacement of the gradient by its Fourier transform

$|\nabla_x \phi|^2 \to q^2 |\phi_q|^2$. Rewriting H in the form of

Harmonic oscillator:

$$H = \int \frac{dq}{2\pi} \left[\frac{\pi_q \pi_{-q}}{2\rho} + \frac{\rho \omega_q^2}{2} \phi_q \phi_{-q} \right],$$ where

now $\omega_q = c|q|$, and $c = \sqrt{T/\rho}$ - is the velocity of sound.

Problem: in the continuum limit $(a \to 0)$ the spectrum $\omega_q = c|q|$ stretches to ∞ with an unbounded # of high frequency/ultraviolet modes.

Introduce a cutoff by writing a small exponential factor in the Fourier transform

$$\phi(x) = \int \frac{dq}{2\pi}\, \phi_q\, e^{iqx}\, e^{-\varepsilon|q|/2}$$

$$\pi(x) = \int \frac{dq}{2\pi}\, \pi_q\, e^{iqx}\, e^{-\varepsilon|q|/2}.$$

Repeating calculation of the commutation relation

$$[\phi(x), \pi(y)] = \int \frac{dq}{2\pi} \frac{dq'}{2\pi}\, e^{i(qx - q'x')} \underbrace{[\phi_q, \pi_{-q'}]}_{2\pi i\hbar\, \delta(q-q')} e^{-\frac{\varepsilon}{2}(|q|+|q'|)} =$$

$$= i\hbar \int \frac{dq}{2\pi}\, e^{iq(x-x') - |q|x} =$$

$$= i\hbar \left[\int_0^\infty dq\, e^{-(\varepsilon - i(x-x'))q} + \int_{-\infty}^0 e^{(\varepsilon + i(x-x'))q} \right] =$$

$$= \frac{i\hbar}{2\pi} \left[\frac{1}{\varepsilon - i(x-x')} + \frac{1}{\varepsilon + i(x-x')} \right] =$$

$$= i\hbar\, \underbrace{\frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + (x-x')^2}}_{\delta_\varepsilon(x-x')}$$

– shows that removing of the ultraviolet modes smears the δ-function into a Lorentzian of width ε.

The regularized Hamiltonian has the form:

$$H = \int \frac{dq}{2\pi}\left[\frac{\pi_q \pi_{-q}}{2\rho} + \frac{\rho\omega_q^2}{2}\phi_q\phi_{-q}\right] e^{-\varepsilon|q|}$$

which has the same form as the discrete lattice, but now the high-momentum modes are cut by the exponential factor, rather than the finite size of the Brillouin zone.

Introduce a, a^+ by

$$\phi_q = \sqrt{\frac{\hbar}{2\rho\omega_q}}\left(a_q + a_{-q}^+\right), \quad \pi_q = -i\sqrt{\frac{\hbar\rho\omega_q}{2}}\left(a_q - a_{-q}^+\right),$$

we find that the creation/annihilation operators satisfy

$$[a_q, a_{q'}^+] = 2\pi\,\delta(q-q').$$

$$\Rightarrow H = \int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{\hbar\omega_q}{2}\left(a_q^+ a_q + a_{-q}a_{-q}^+\right)e^{-\varepsilon|q|}$$

$$\Rightarrow H = \int_{-\infty}^{\infty}\frac{dq}{2\pi}\hbar\omega_q\left(\hat{n}_q + \overbrace{2\pi\delta(0)}^{L}\cdot\frac{1}{2}\right)e^{-\varepsilon|q|}$$

which means that ~~the~~ zero-point energy scales with length:

$$E_{g.s.} = L\int_0^{\infty}\frac{dq}{2\pi}\hbar c|q|e^{-\varepsilon|q|} = \frac{L\hbar c}{2\pi\varepsilon^2}$$

So if we remove momentum cutoff zero-point energy per unit length becomes unbounded/infinite in the continuum limit.

Now, for phonons we have that

$$H_{phonon} = \sum_{\vec{k}} \hbar\omega(k)\left[n_k + \tfrac{1}{2}\right], \quad \vec{n}_q = \hat{b}_k^+ b_k -$$

- represents the phonon number operator.

Here, one may choose displacement operators, $\hat{\phi}_q$ as the operators of the free field.

However it is convenient to define these operators differently, based on electron-phonon interaction in metals: Define

$$\tilde{\varphi}(x) = i f \sum_{\vec{k}} \sqrt{\frac{\hbar s\omega_k}{2}} \left\{ b_k e^{i[\vec{k}\vec{r} - \omega_{\vec{k}} t]} - b_k^+ e^{-i[\vec{k}\vec{r} - \omega_{\vec{k}} t]} \right\},$$

which follows from the Debye model of longitudinal phonons, and summation over $\vec{k}$ is restricted by $|\vec{k}| < k_D$:

$$H_{el-ph} = \underbrace{g}_{\text{interaction constant}} \int C_\alpha^+(\vec{r})\, c_\alpha(\vec{r})\, \varphi(\vec{r})\, d\vec{r}$$

The Green's function of the phonon is

$$D(x,x') = -i \langle T(\tilde{\varphi}(x), \tilde{\varphi}(x')) \rangle .$$

Using the expression for free-field operators $\tilde{\varphi}(x)$ and the fact that there are no phonons in the ground state $b|0\rangle = 0$, we obtain

$$D^{(0)}(\cancel{k,\omega}) = i \frac{\omega_k^2(\vec{k})}{\omega^2 - \omega_0^2(\vec{k}) + i\delta} \quad \text{or} \quad \omega_{\vec{k}} \equiv \omega(\vec{k}).$$

$$D^{(0)}(x) = -\frac{i}{V} \sum_{\vec{k}} \frac{\omega_{\vec{k}}}{2} \left[\frac{1}{\omega - \omega_{\vec{k}} + i\delta} - \frac{1}{\omega + \omega_{\vec{k}} - i\delta} \right]$$

Bound states of two particles:

Consider an "immediate" interaction potential, that does not have any retardation:

$$U_{12} = U(\vec{r}_1 - \vec{r}_2) \cdot \delta(t_1 - t_2).$$

Then, in momentum representation, the interaction Hamiltonian is given by

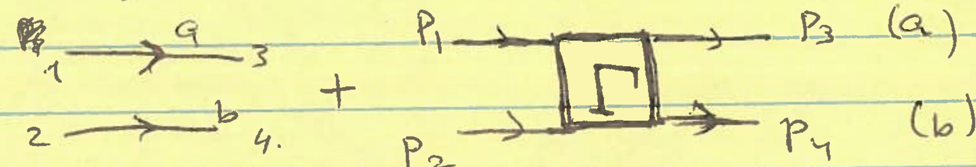
$$H_{int} = (2\pi)^4 \int C^+_{p_1} C_{p_3} C^+_{p_2} C_{p_4} U(k)\, \delta(p_1 - p_3 + k) \cdot \delta(p_2 - p_4 - k) \times d^4 k$$

This is Fourier transformed form of

$$H_{int} = \iint C^+_{r_1} C_{r_1} U(\vec{r}_1 - \vec{r}_2) C^+_{r_2} C_{r_2},$$

As before, we define the interaction vertex Γ from the two-particle Green's function

$K_{ab} =$



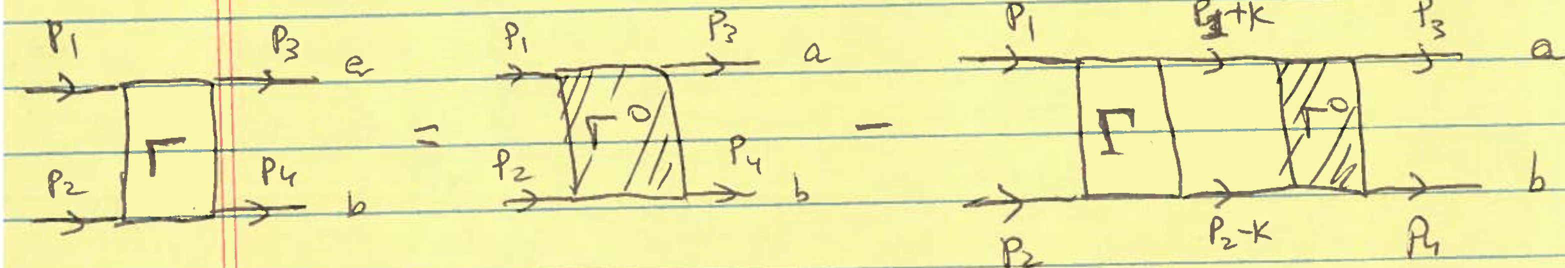
$$K_{1-2;\,3-4} = (2\pi)^8 \delta_{p_1, p_3} \delta_{p_2, p_4} G_0^{(a)}(p_1)\, G_0^{(b)}(p_2) +$$

$$+ (2\pi)^4 \cdot G_0^{(a)}(p_1)\, G_0^{(b)}(p_2)\, G_0^{(a)}(p_3)\, G_0^{(b)}(p_4)\; i\Gamma_{p_1 p_2 p_3 p_4} \delta_{p_1 + p_2;\, p_3 + p_4}$$

Interaction vertex $\Gamma_{p_1 p_2; p_3 p_4}$ can be found from Bethe-Selpeter equations

$$\Gamma_{1,2,3,4} = \Gamma^{0}_{1,2,3,4} + i \int \Gamma^{0}_{\cancel{1,2,}\vec{p}_1,\vec{p}_2,\vec{p}_1+\vec{k},\vec{p}_2-\vec{k}}\, G_0(\vec{p}_1+\vec{k})\, G_0(\vec{p}_2-\vec{k})$$

$$\times\, \Gamma_{p_1+k,\, p_2-k,\, p_3,\, p_4} \cdot \frac{d^4 k}{(2\pi)^4}$$

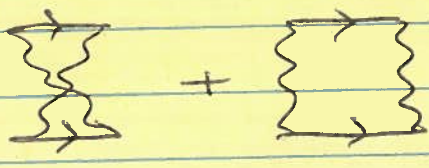


→ full interaction vertex

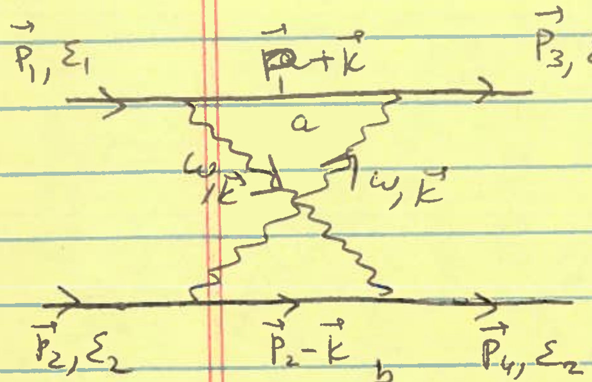
– the sum of irreducible interaction vertices

When there is no retardation, $U(\omega,\vec{k}) = U(\vec{k})$,

~~th~~ Let us consider ~~second order~~ the second order in interactions corrections to Γ:

$\Delta\Gamma^{(2)} =$  .

The first diagram gives



$$= g^2 \int \frac{d^3\vec k}{(2\pi)^3}\,\frac{d\omega}{2\pi} \times$$

$$\times\, G_0^{(a)}(\varepsilon_1-\omega, \vec p_1+\vec k)\, G_0^{(b)}(\varepsilon_2+\omega, \vec p_2-\vec k)\, U(\omega,\vec k)\cdot U(\omega,k) =$$

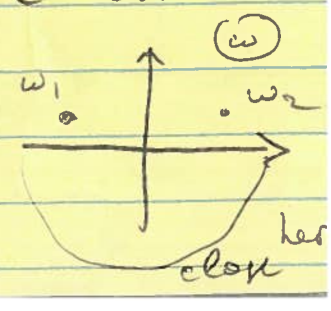
$$= g^2 \int \frac{d^3k}{(2\pi)^4}\, U(\vec k)\cdot U(k) \underbrace{\int d\omega\, G_0(\varepsilon_2+\omega, \vec p_2-\vec k)\, G_0(\varepsilon_1-\omega, \vec p_1+\vec k)}_{\text{calculate this}}$$

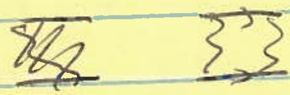
$$\int d\omega\, G_0^{(a)}(\varepsilon_1-\omega, \vec p_1+\vec k)\, G_0^{(b)}(\varepsilon_2+\omega, \vec p_2-\vec k) =$$

$$= \int \frac{d\omega}{\left(\omega-\left[\varepsilon_1-\frac{(\vec p_1+\vec k)^2}{2m}+i\delta\right]\right)\left(\omega-\left[\varepsilon_2-\frac{(\vec p_2-\vec k)^2}{2m}+i\delta\right]\right)} = 0$$

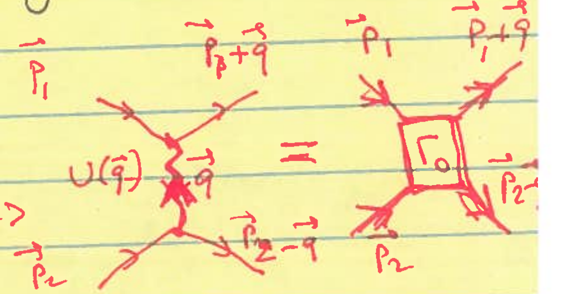
because both poles $\omega=\omega_1$, $\omega=\omega_2$ are on the upper half-plane

$$\omega_1 = \varepsilon_1 - \frac{(\vec p_1+\vec k)^2}{2m} + i\delta\,, \quad \omega_2 = \varepsilon_2 - \frac{(\vec p_2-\vec k)^2}{2m} + i\delta\,,$$



The ~~second~~ second order diagram: , is <u>not</u> $=0$, since the poles of two electron Green's functions are located in different half-planes. However,  does not contribute into Γ^0, as it is reducible. Since ALL irreducible diagrams contain crossing wavy lines $\Rightarrow$ we conclude that

$$\Gamma^0_{p_1,p_2,p_1+q,p_2-q} = U(\vec{q}). \Rightarrow$$



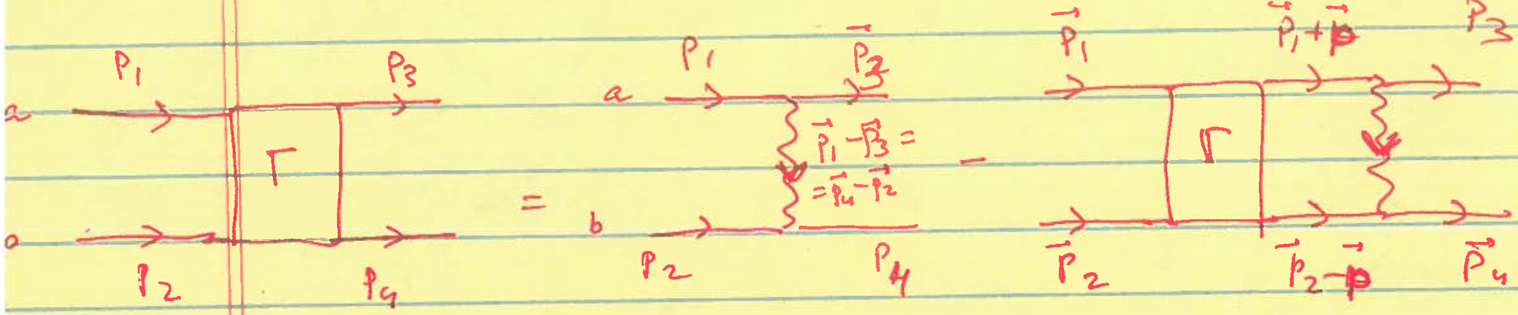
Substituting this expression of Γ^0 into Bethe-Salpeter equation we see that since Γ^0 does not depend on ω $\Rightarrow$ Γ does not depend on ω either.

So ω dependence is in Green's functions only, and therefore one can exactly perform ω-integration:

$$i\int G_0^{(a)}(\varepsilon_1-\omega)\, G_0^{(b)}(\varepsilon_2+\omega)\,\frac{d\omega}{2\pi} = \frac{1}{\varepsilon_1+\varepsilon_2-\frac{(p_1+k)^2}{2m_a}-\frac{(p_2-k)^2}{2m_b}+i\delta}$$

$$\left\{\begin{matrix} m_a = m_1 \\ m_b = m_2 \end{matrix}\right| .$$

Here, one can decompose the denominator of Bethe-Salpeter equation by introducing center-of-mass motion and relative motion:



Introduce relative momenta (to C.M.)

$$\vec{k} = \frac{m_2\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}, \quad \vec{k}' = \frac{m_2\vec{p}_3 - m_1\vec{p}_4}{m_1+m_2}, \quad \frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2} = \frac{p_3^2}{2m_1}+\frac{p_4^2}{2m_2}$$

$$\Rightarrow \quad \vec{k}-\vec{k}' = \vec{p}_1-\vec{p}_3 = \vec{p}_4-\vec{p}_2 \quad \text{and} \quad \frac{k^2}{2\mu} = \frac{k'^2}{2\mu} = \Omega_0,$$

where $\mu = \dfrac{m_1 m_2}{m_1+m_2}$, $M = m_1+m_2$.

Then $U(\vec{p}_1-\vec{p}_3) = U(\vec{p}_4-\vec{p}_2) = U(\vec{k}-\vec{k}')$ and

Bethe-Salpeter equation reads:

does not depend on full $\vec{P}$

$$\Gamma(k,k') = U(\vec{k}-\vec{k}') + \int \frac{d^3\vec{k}_{int}}{(2\pi)^3} U(-\vec{k}_{int}) \Gamma_{p_1+k_{int}, p_2-k_{int}, p_3, p_4} \times$$

$$\times \frac{1}{\varepsilon_1+\varepsilon_2 - \frac{(\vec{p}_1+\vec{k}_{int})^2}{2m_1} - \frac{(\vec{p}_2-\vec{k}_{int})^2}{2m_2} + i\delta}$$

Using the center of mass variables

$$\frac{(\vec{p}_1+\vec{k}_{int})^2}{2m_1} + \frac{(\vec{p}_2-\vec{k}_{int})^2}{2m_2} = \frac{(\vec{p}_1+\vec{p}_2)^2}{2M} + \frac{(\vec{k}+\vec{k}_{int})^2}{2\mu}.$$

where $\vec{k} = \dfrac{m_2\vec{p}_1 - m_1\vec{p}_2}{m_1+m_2}$.

Changing integration variable from $\vec{k}_{int}$ to

$\vec{q} = \vec{k}_{int} + \vec{k}$, we obtain $\vec{k}_{int} = \vec{q}-\vec{k}$

$$\Gamma(\vec{k},\vec{k}') = U(\vec{k}-\vec{k}') + \int \frac{U(\vec{k}-\vec{q})\Gamma(\vec{q},\vec{k}')}{\underbrace{\varepsilon_1+\varepsilon_2}_{\Omega=\varepsilon_3+\varepsilon_4} - \frac{(P_1+P_2)^2}{2M} - \frac{q^2}{2\mu} + i\delta} \frac{d^3q}{(2\pi)^3}$$

or

$$\Gamma(\vec{k},\vec{k}') = U(\vec{k}-\vec{k}') + \int \frac{U(\vec{k}-\vec{q})\,\Gamma(\vec{q},\vec{k}')}{\Omega_0 - \frac{q^2}{2\mu} + i\delta} \frac{d^3q}{(2\pi)^3}$$

$$\Omega_0 = \Omega - \frac{(P_1+P_2)^2}{2M} = \varepsilon_1+\varepsilon_2 - \frac{(P_1+P_2)^2}{2M}$$

Notice that on the $\boxed{\text{mass shell}}$, when

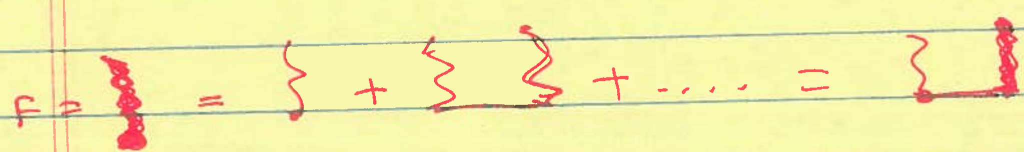
$$\varepsilon_1 = \frac{p_1^2}{2m_1}\,, \quad \varepsilon_2 = \frac{p_2^2}{2m_2}$$

$$\Omega_0 = \varepsilon_1 + \varepsilon_2 - \frac{(p_1+p_2)^2}{2M} \longrightarrow \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - \frac{(p_1+p_2)^2}{2M} = \frac{K^2}{2\mu} = \frac{K'^2}{2\mu}.$$

$$\mu = \frac{m_1 m_2}{m_1+m_2}$$ is the reduced mass.

Then, the equation we obtained for $\underline{\underline{\Gamma}}$ is ~~eat~~ exactly coinciding with an equation for scattering cross ~~secti~~ amplitude $F(\vec{K}, \vec{K}')$ of a single particle with mass ~~[illegible]~~ μ.

$$F = \hat{U} + \hat{U}\hat{G}_0\hat{U} + \hat{U} G_0 U G_0 U + \ldots = \hat{U} + \hat{U}\hat{G}_0\hat{F}$$



is a single-particle problem of scattering (on mass shell).