Homework #2:

P1: At finite temperature, Fermi-distribution function is smeared by a scale $\sim T$, which corresponds to the uncertainty of the radius of the Fermi-sphere

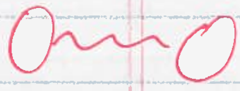
$$\delta p_0 \approx T/v_F.$$

The spatial oscillations of electron density with $k = 2k_F$ are thus affected by T.

Derive ~~Consider~~ Friedel oscillations at finite-T, $T \ll E_F$. Show that the oscillations persist at distances $r < l_T$, where $l_T = \frac{\hbar v_F}{2\pi T}$ is the thermal length. Show also that at $r \geq l_T$, the oscillations are exponentially damped.

P2: (Electron interaction energy in a metal): At high densities, a metal behaves almost like an ideal Fermi-gas. Coulomb-interaction effects in it can be estimated perturbatively, upon expansion in interactions.

First order corrections to the interaction energy are given by

 Hartree diagram

 Fock diagram.

It can be shown that the first (Hartree) diagram does not contribute to the interaction energy (as it is being compensated).

~~Therefore~~ Calculate the correction coming from Fock diagram (exchange energy).

Show that it scales with density of electrons ~~as~~ in 3D as:

$$\Delta E_{exch} \approx n^{4/3}.$$

Hint!
$$\Delta E_{exch} = -\frac{1}{2} T^2 \lim_{\substack{\tau_1 \to 0 \\ \tau_2 \to 0}} \sum_{\varepsilon_1, \varepsilon_2} \sum_{p_1, p_2} G(i\varepsilon_1, p_1)\, G(i\varepsilon_2, p_2) \times$$
$$\times V_{\vec{p}_1 - \vec{p}_2}\, e^{i\varepsilon_1 \tau_1 + i\varepsilon_2 \tau_2},$$

where $G(i\varepsilon, \vec{p}) = \dfrac{1}{i\varepsilon - \xi_p}$ - electron Green's function.

$$V_k = \frac{4\pi e^2}{k^2}.$$

P3: Find the statistics of quasi-holes in the $\nu = 1/m$ Laughlin state:

$$\Psi\left(\{z,\bar z\} \mid \eta_1 \ldots \eta_M\right) = \prod_{j=1}^{M} \prod_{i=1}^{N} (z_i - \eta_j) \prod_{k<l} (z_k - z_l)^m e^{-\sum_{i=1}^{N} \frac{|z_i|^2}{4\ell_B^2}}.$$

P4: Show that the Chern-Simons equations of motion $F_{\mu\nu} = \frac{1}{\kappa} \varepsilon_{\mu\nu\rho} J^\rho$

have a solution determining the gauge field $A_\mu(\vec r, t)$ as follows (in the gauge

when $\vec\nabla \cdot \vec A = 0$ and $A_0 = 0$):

$$A^i(\vec r, t) = \frac{e}{2\pi\kappa} \sum_a \varepsilon^{ij} \frac{x^j - x_a^j(t)}{|\vec r - \vec r_a(t)|^2}.$$

* Show that the same vector potential can be written as:

$$A_i(\vec r) = \frac{e}{2\pi\kappa} \sum_{a=1}^{N} \partial_{r_i} \arg\left[\vec r - \vec r_a\right].$$